Improving the Energetic Reasoning: How I followed 15-year-old advice from my supervisor

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Purposes of this talk

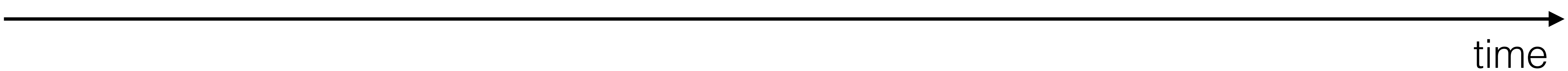
- To reveal some of my supervisor’s greatest advice.
- To show how I still apply his advice when working my students.
- To present a $O(n \log^2 n)$ checker for the energetic reasoning

Yanick Ouellet

Outline

- The CUMULATIVE constraint
- The energetic check
- Our new checker
  - The computation of energy (Advice #1)
  - Monge matrices (Advice #2)
  - Experiments (Advice #3)
- A last advice (Advice #4)
- Conclusion

Definitions



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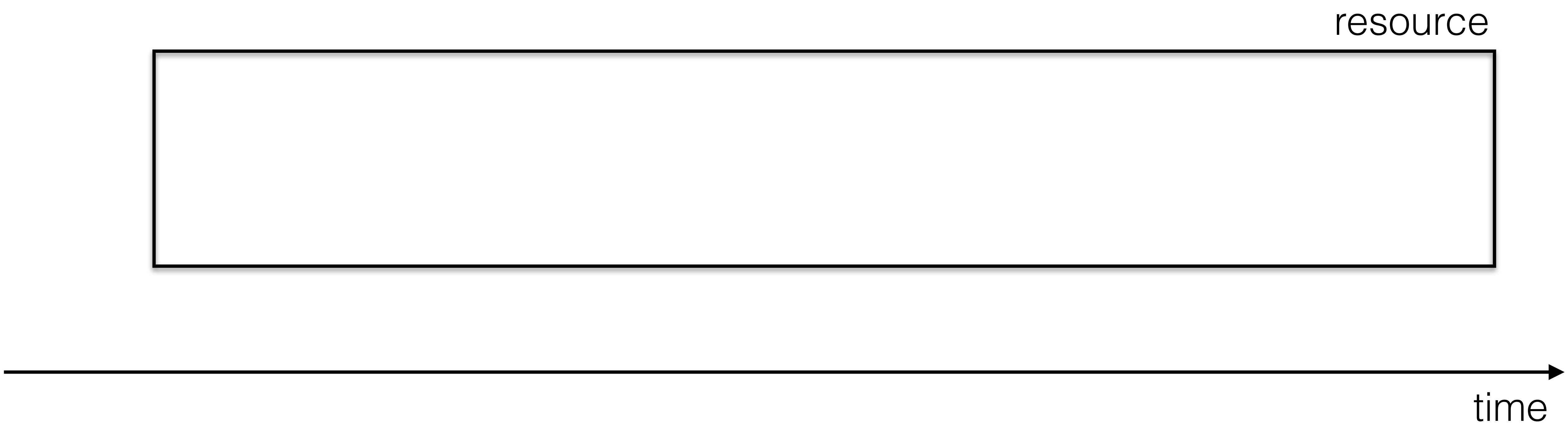

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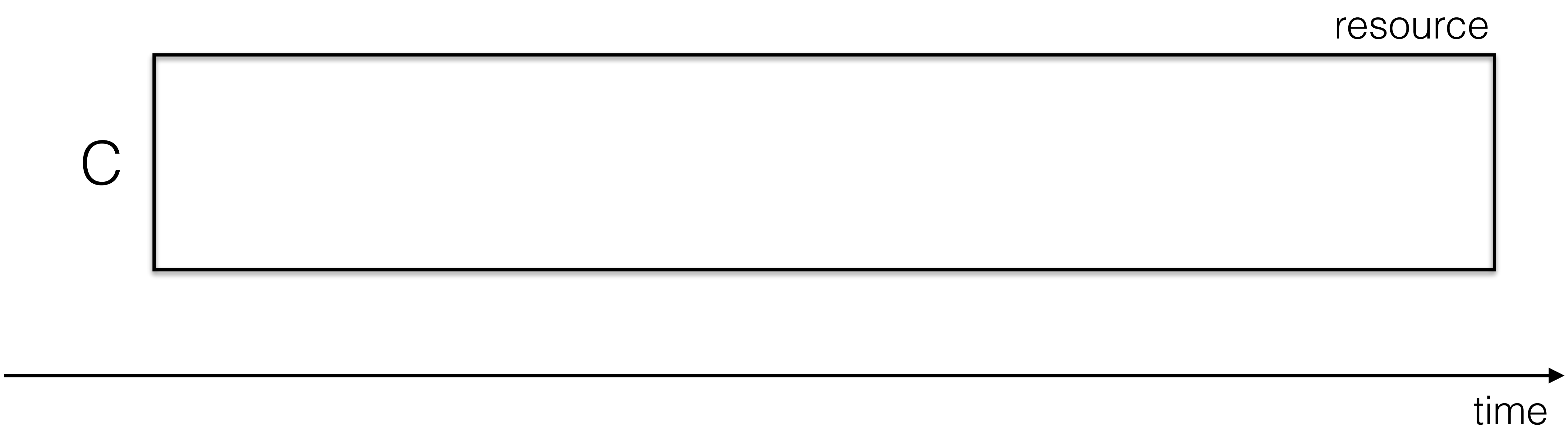

- C: capacity of the resource

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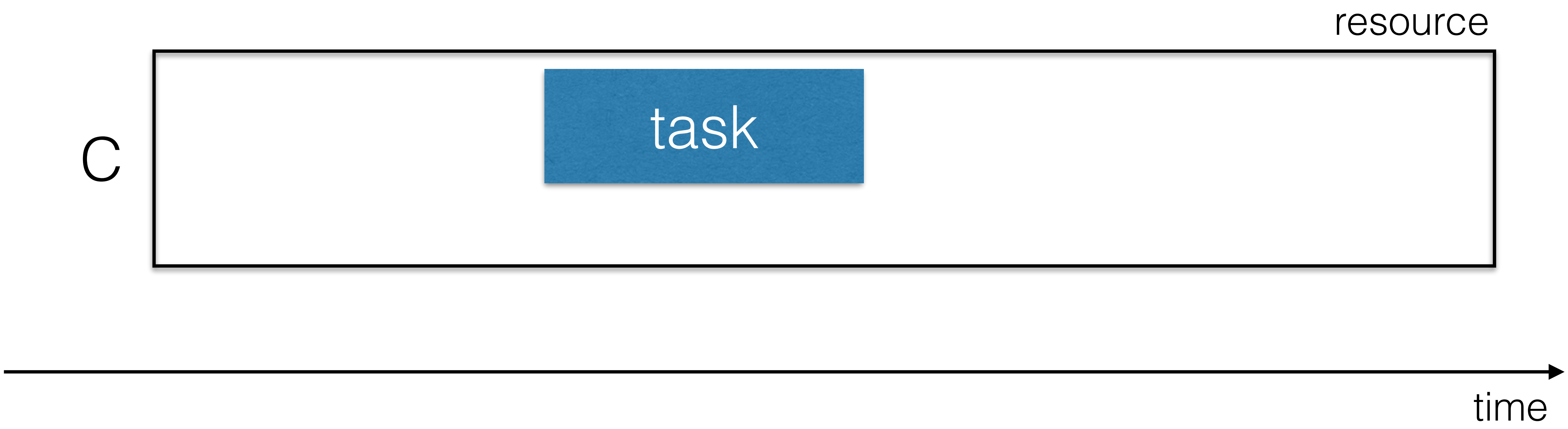

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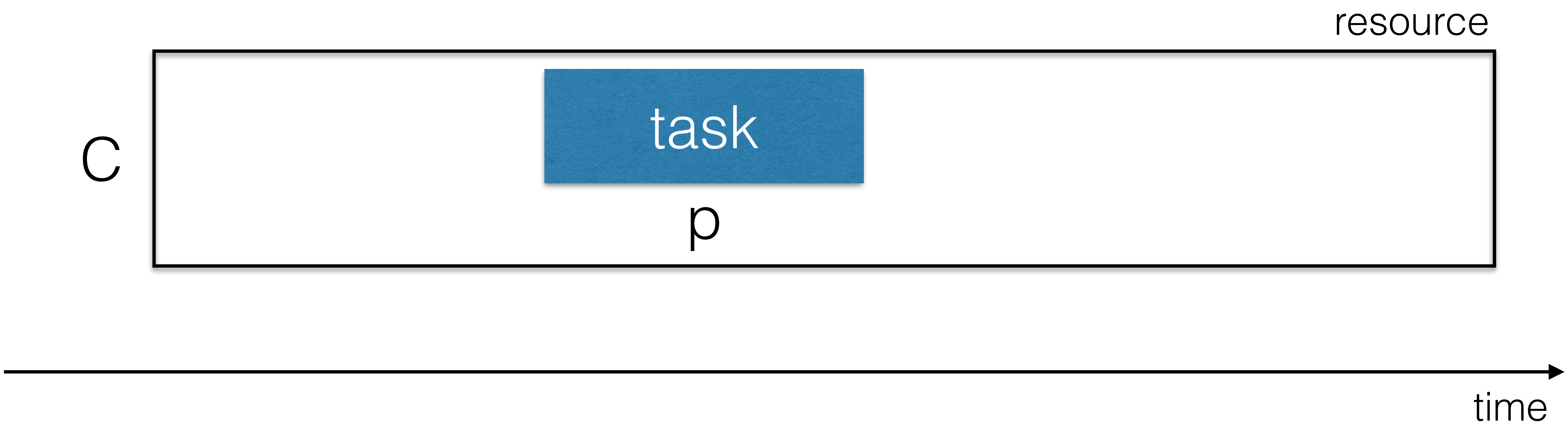

- C: capacity of the resource
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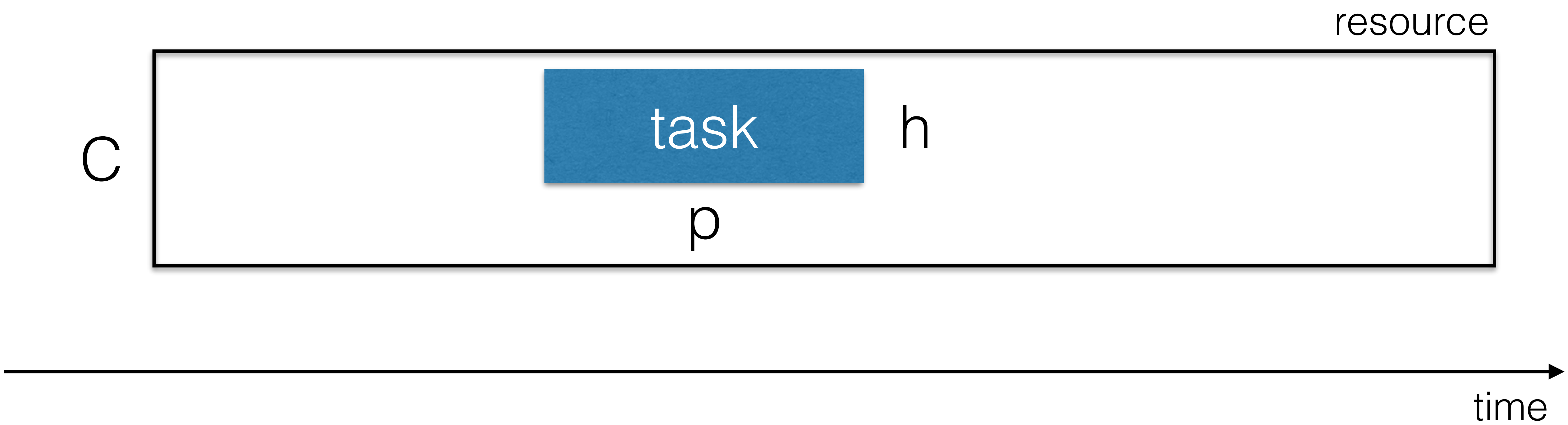

- C: capacity of the resource
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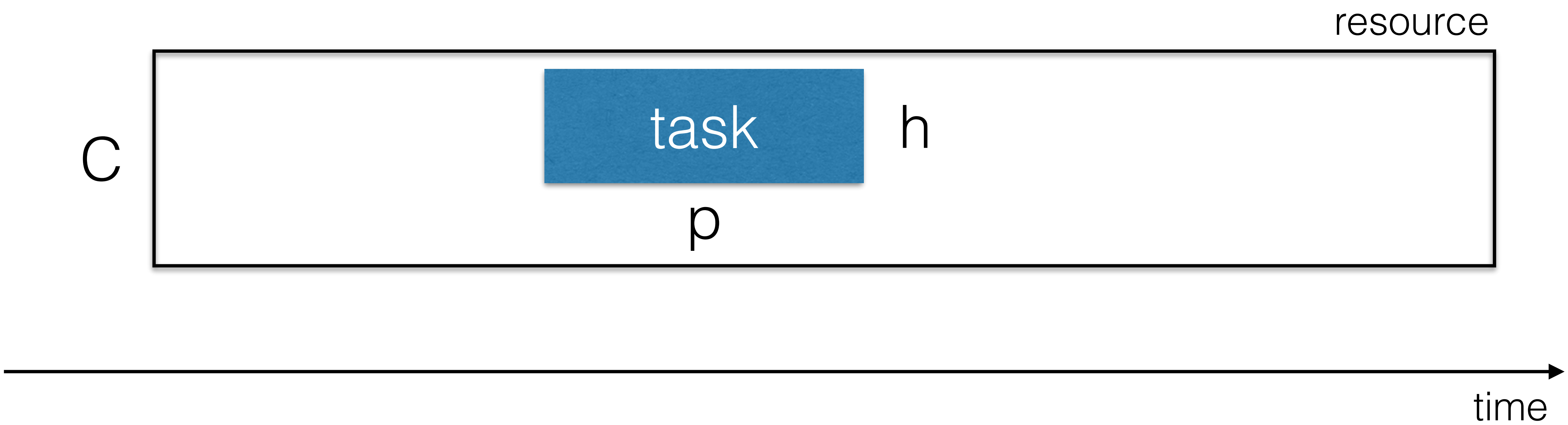

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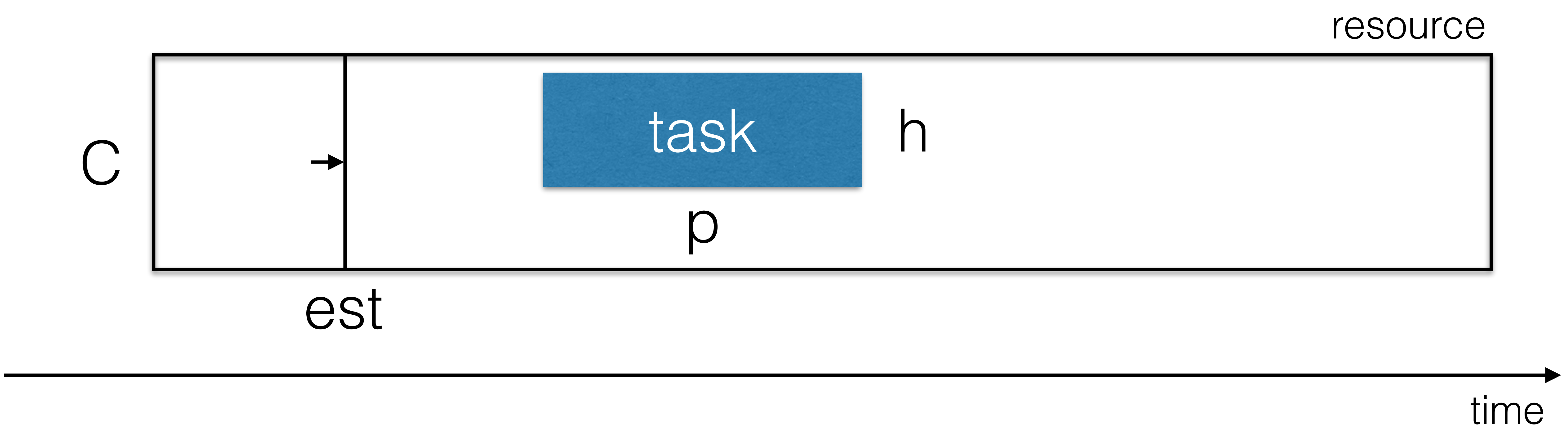

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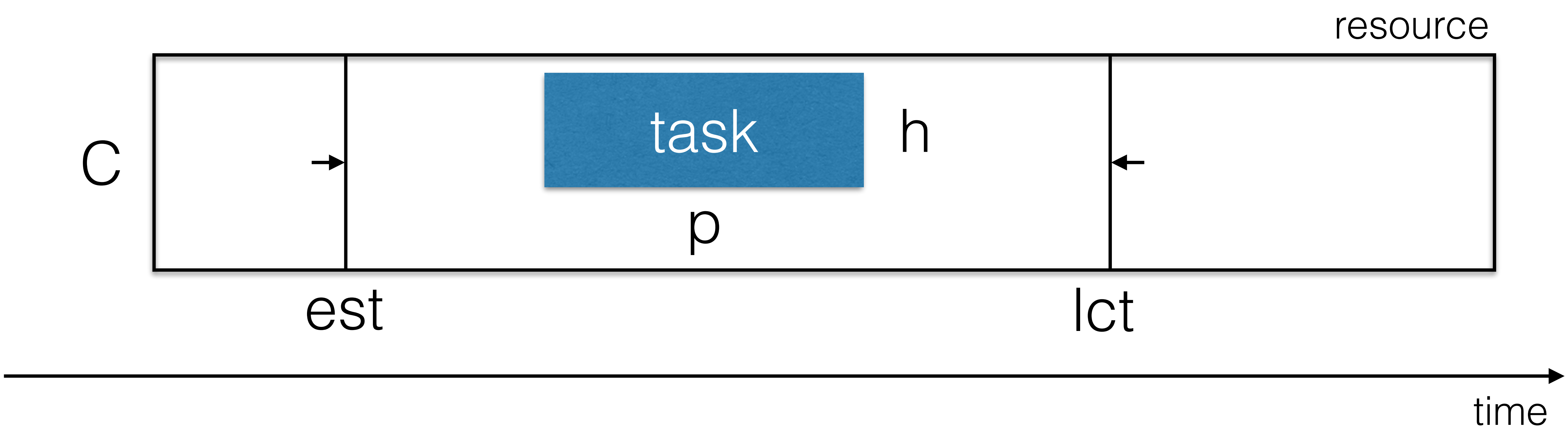

- C: capacity of the resource
- p: processing time
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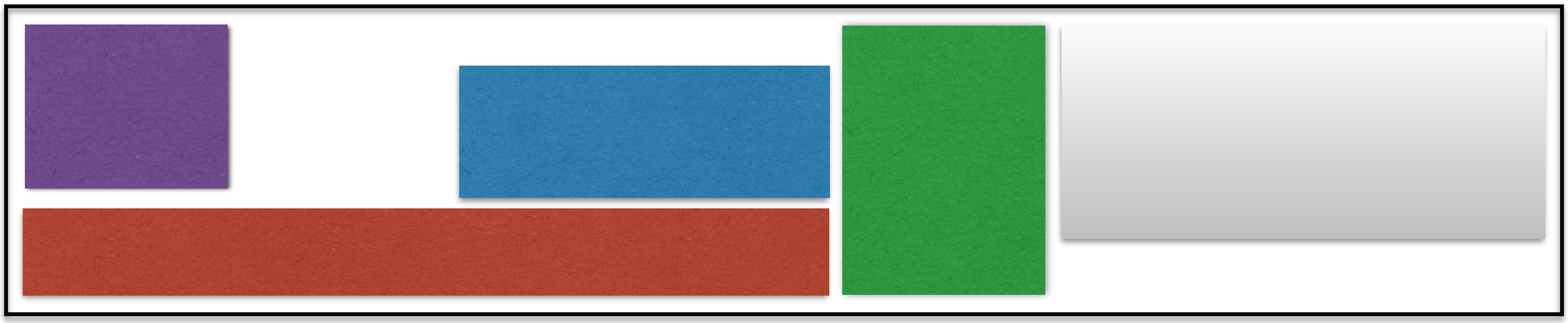
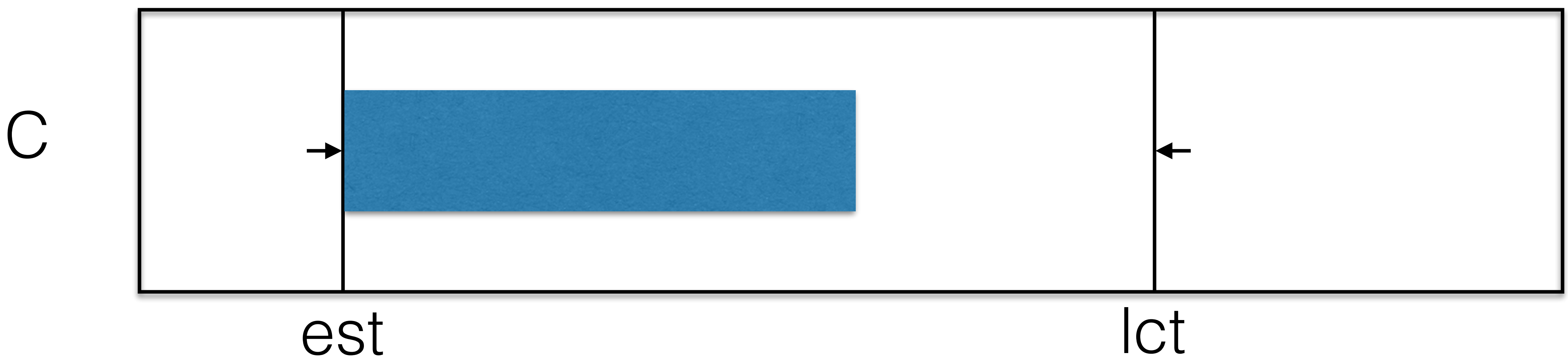
- e: energy ($e = p \times h$)
- est: earliest starting time
- lct: latest completion time

The CUMULATIVE constraint



- Tasks must be scheduled between their est and lct.
- No overlap.
- The capacity of the resource is not exceeded.

The energetic check



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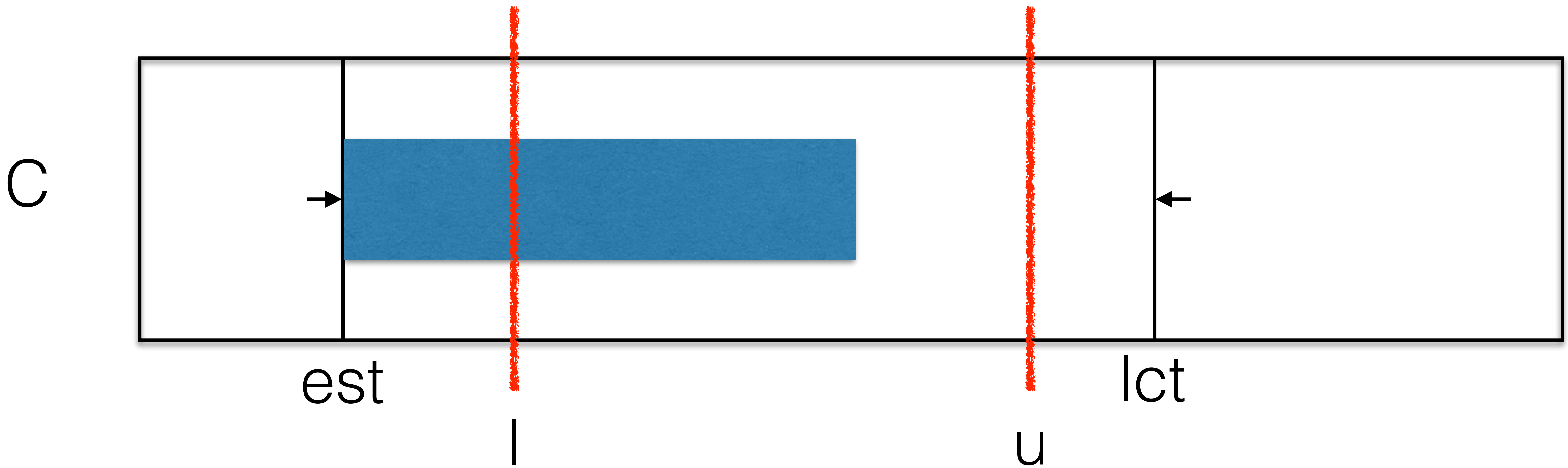

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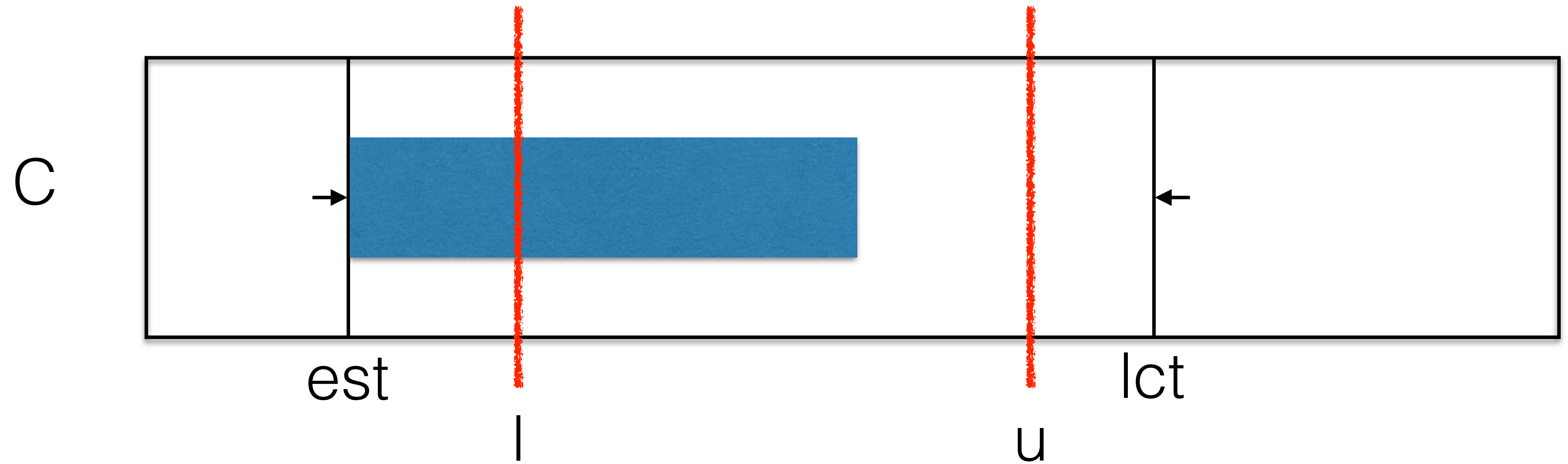

$$E(i, l, u) = h_i \cdot \max(0, \min(\qquad\qquad))$$

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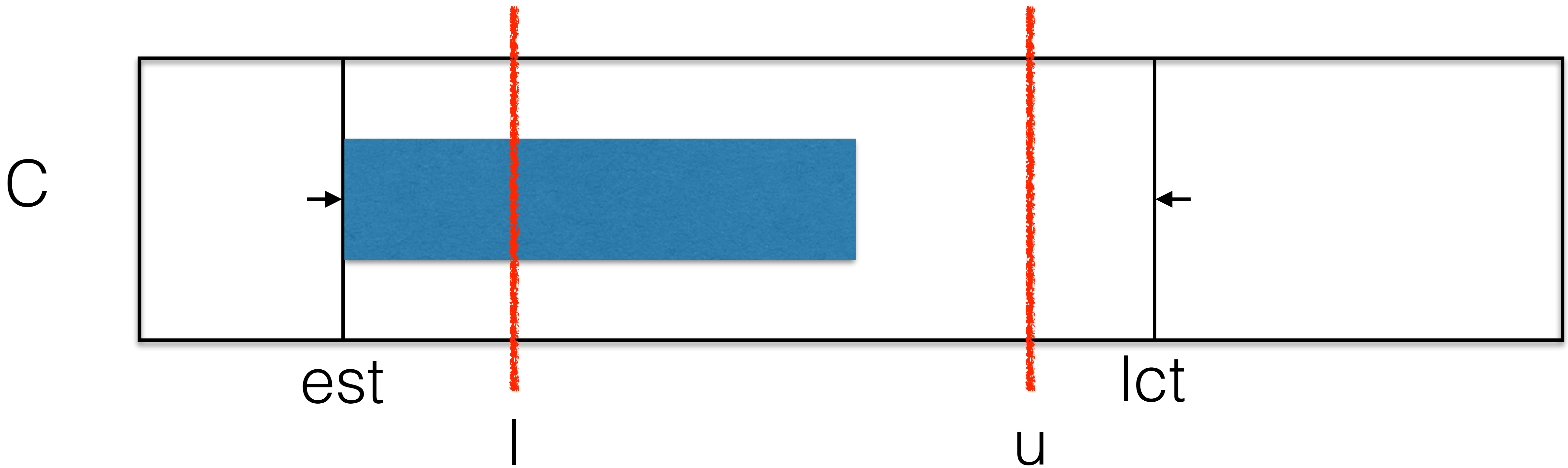

$$E(i,l,u) = h_i \cdot \max(0, \min(u-l, \qquad\qquad\qquad\qquad))$$

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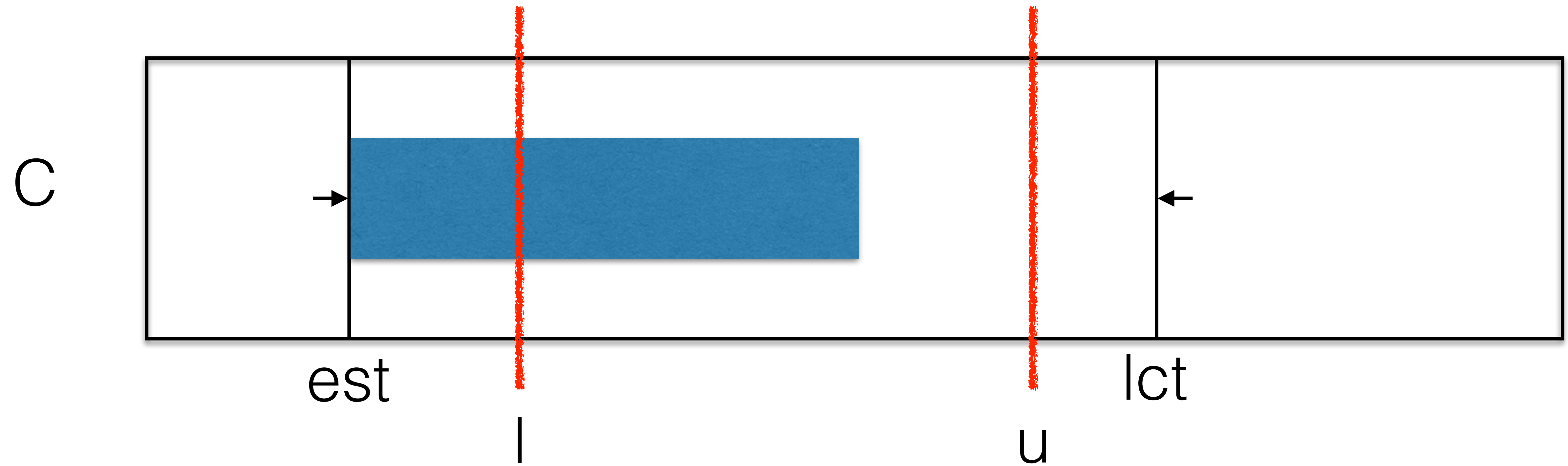

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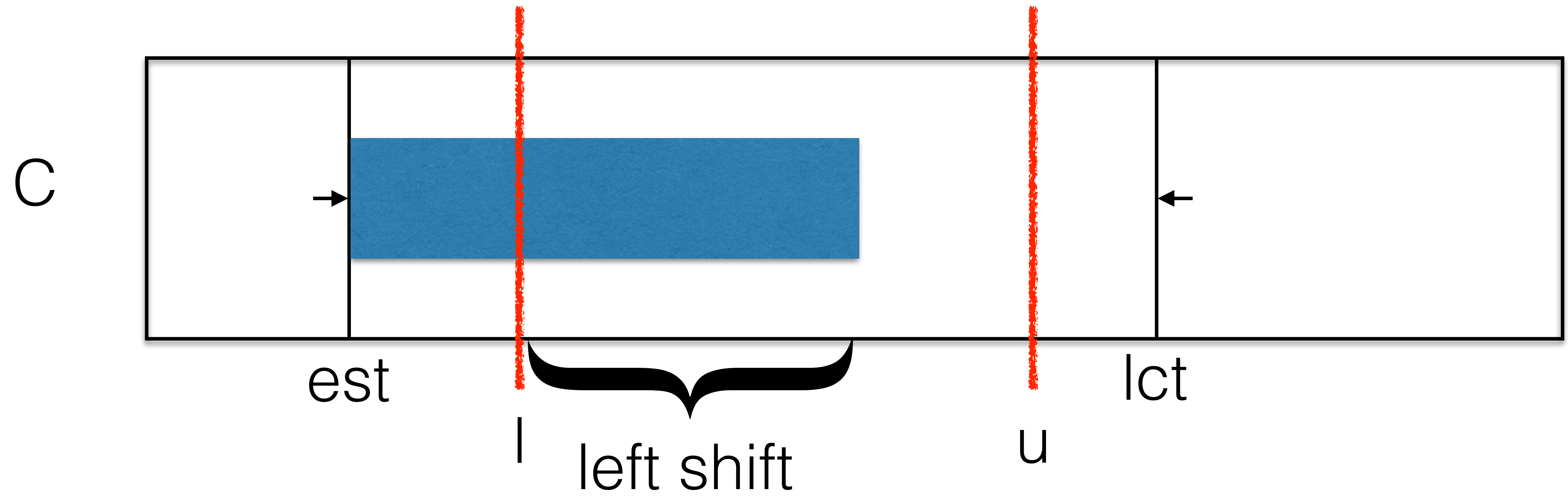

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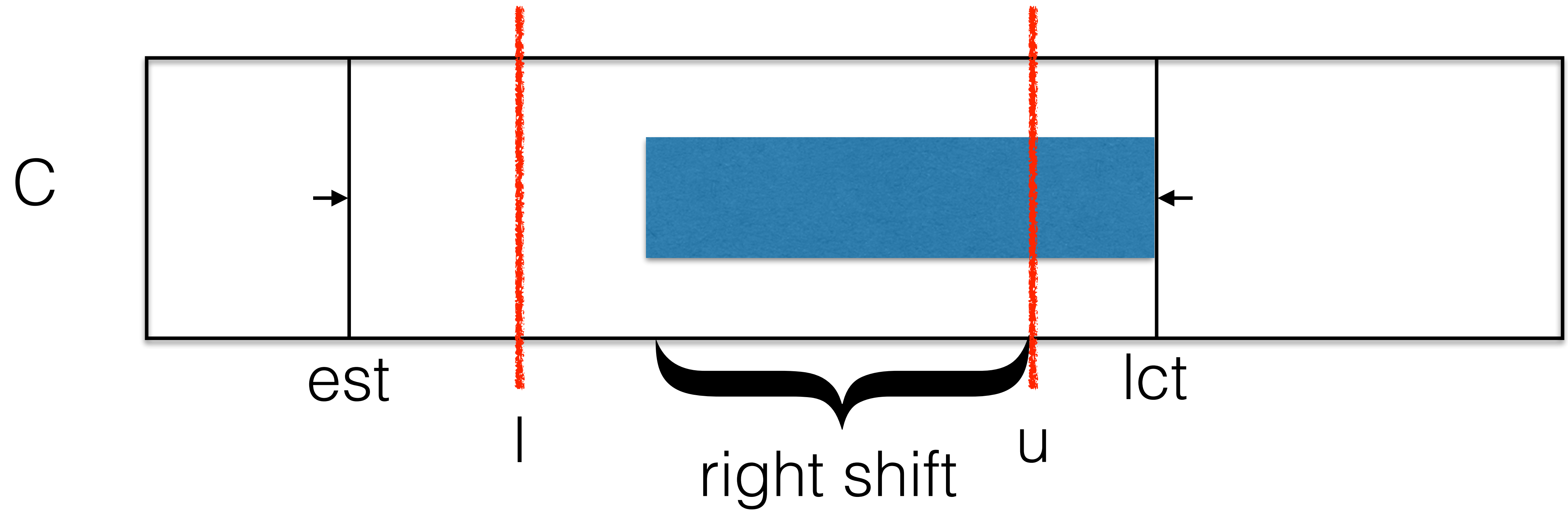

$$E(i,l,u) = h_i \cdot \max(0, \min(u - l, p_i, \mathrm{est}_i + p_i - l, u - (\mathrm{lct}_i - p_i)))$$

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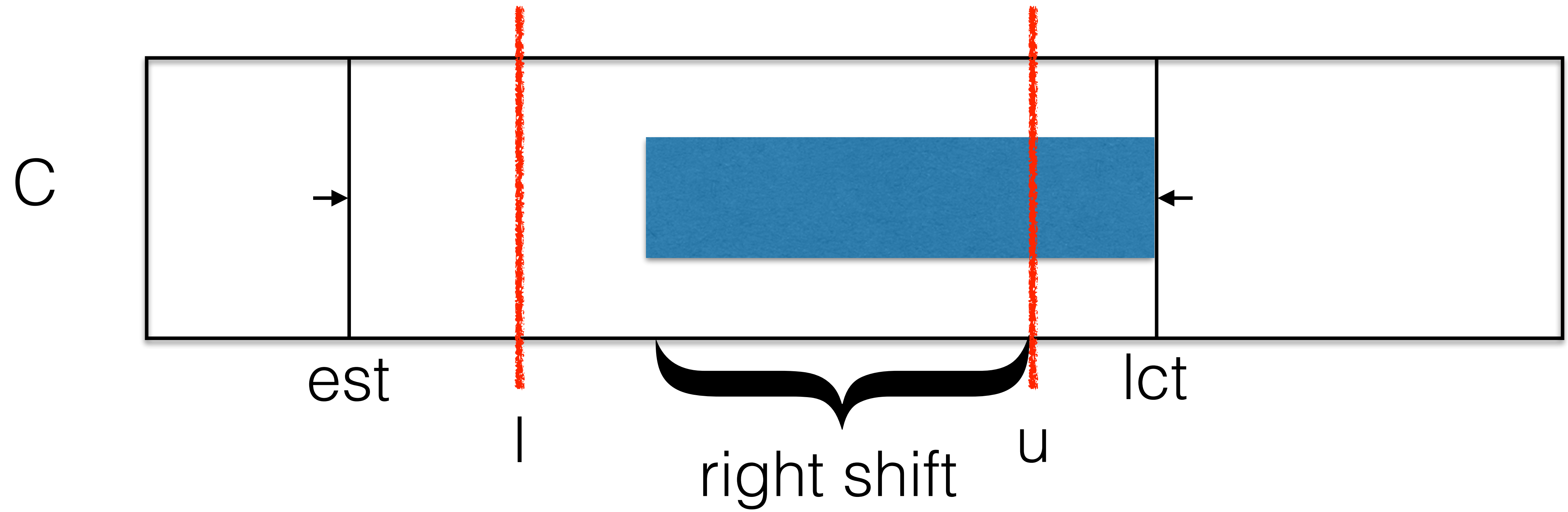

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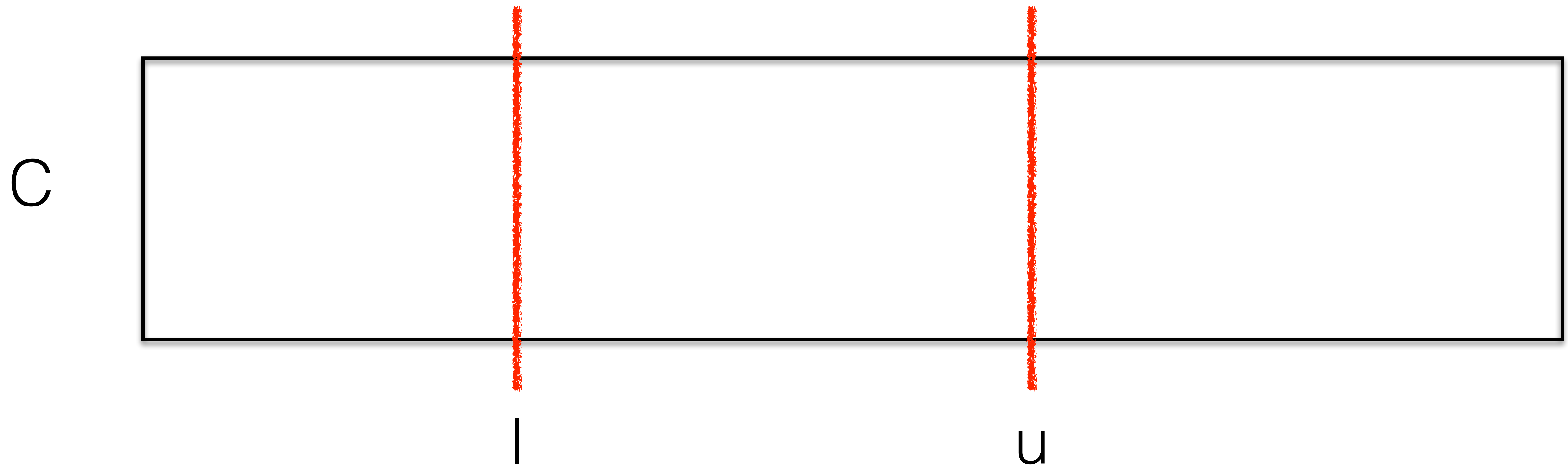

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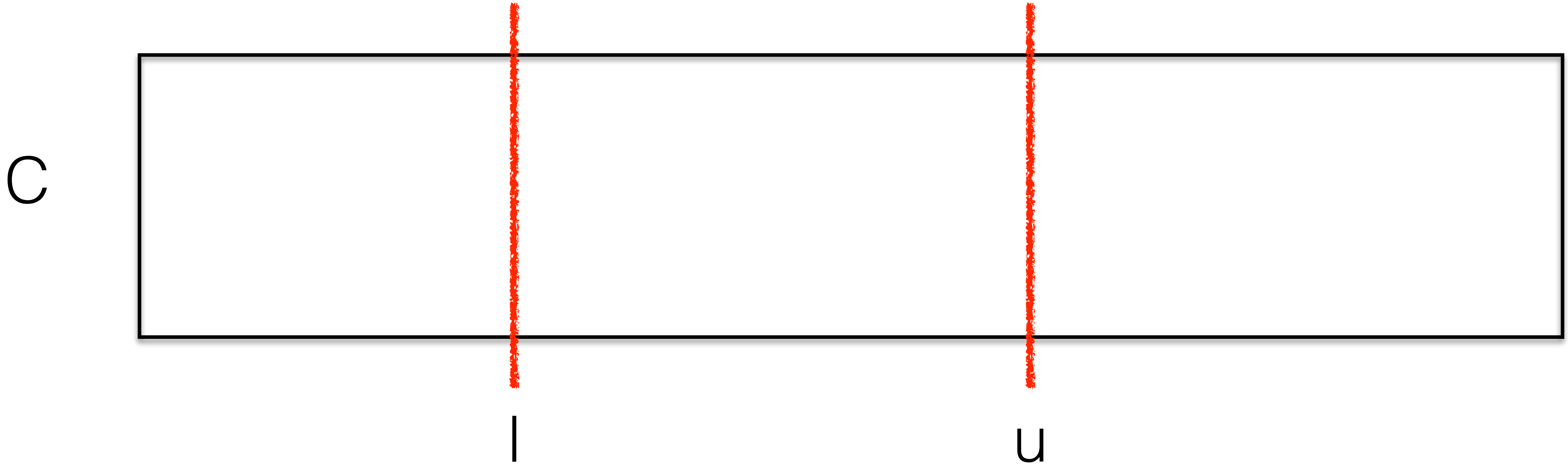

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$$S(l,u) = C \cdot (u-l) - \sum_i E(i,l,u) \geq 0$$

Existing checkers

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- The slack $S(l, u)$ is computed in constant time, using the previous computation of $S(l, u-1)$.
- Running time complexity: $O(n^2)$
- Derrien and Petit reduced the multiplicative constant by a factor of 7.

Goal

- To perform the energetic check in sub-quadratic time.
  - We need to test fewer than $O(n^2)$ intervals
  - We will need to compute the slack, upon request, for any time interval [l, u)

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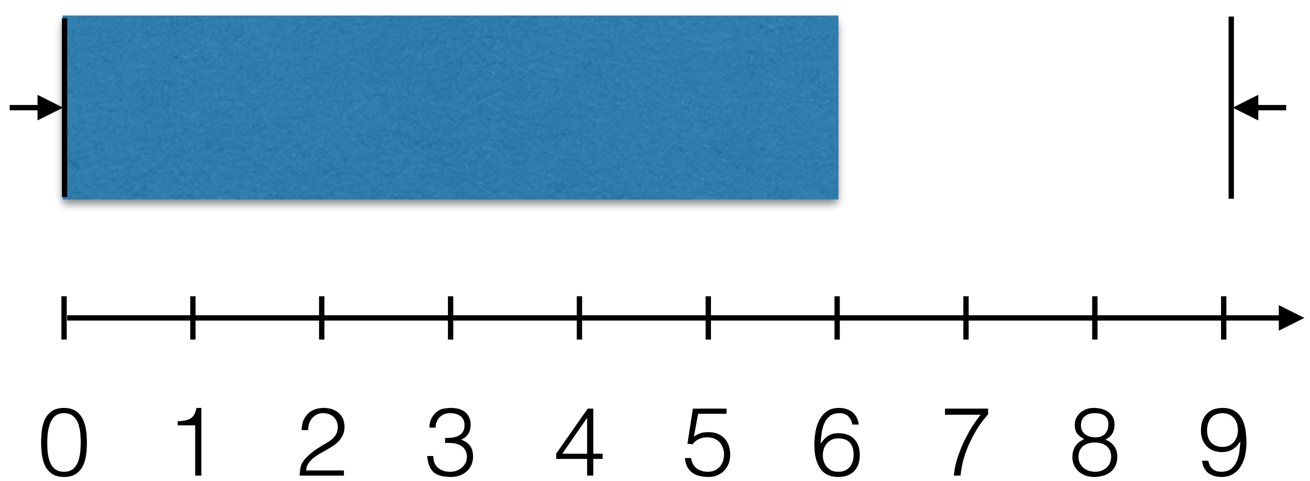
Let's start by solving this problem

Advice #1

Reduce your problem to one that is already solved.

- Reductions can provide solutions out of the box.
- If not, they give a direction how to adapt a solution to your problem.
- Take time to reformulate your problem using different abstractions: graphs, points/vectors, …

Reduction



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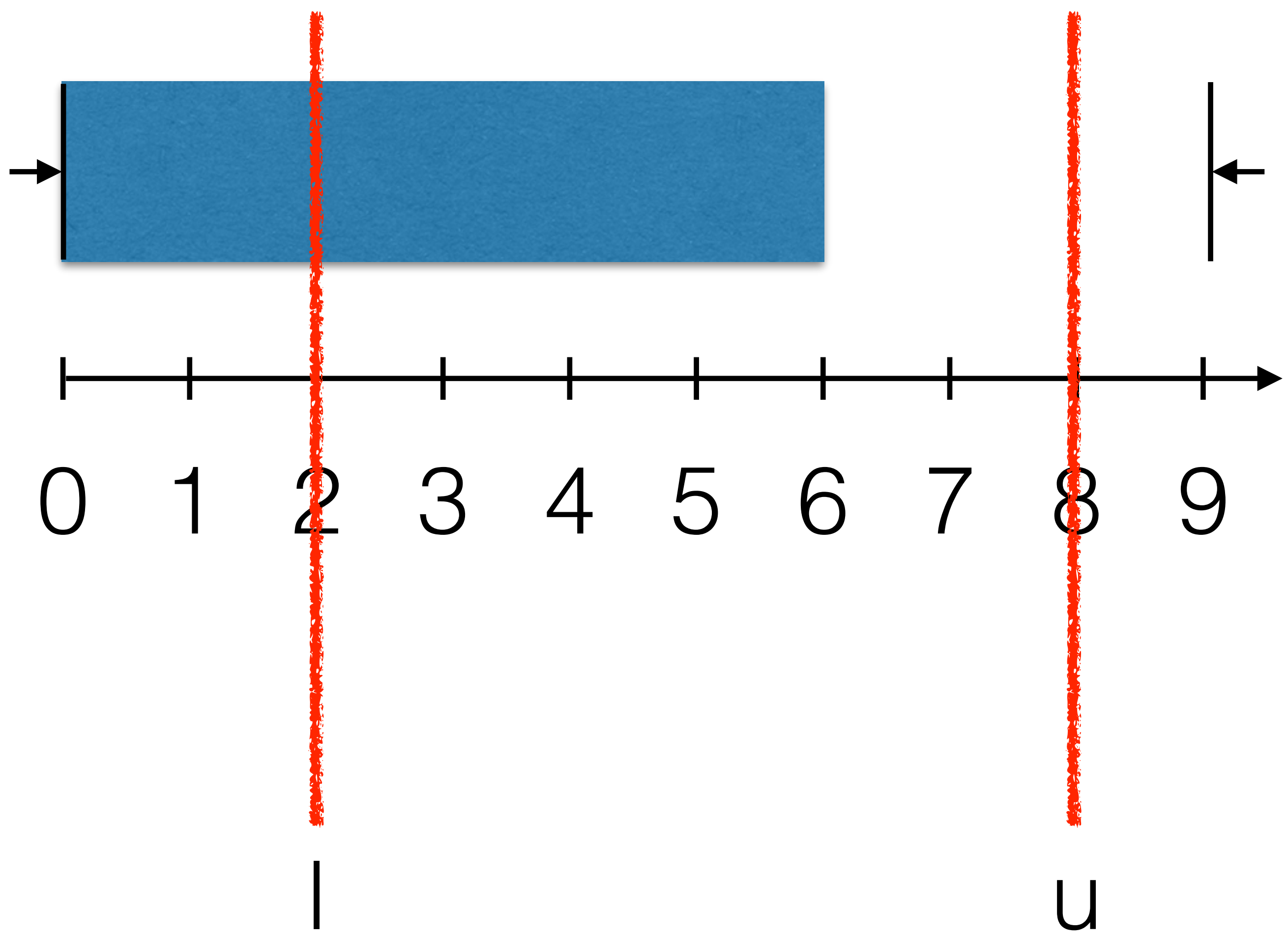

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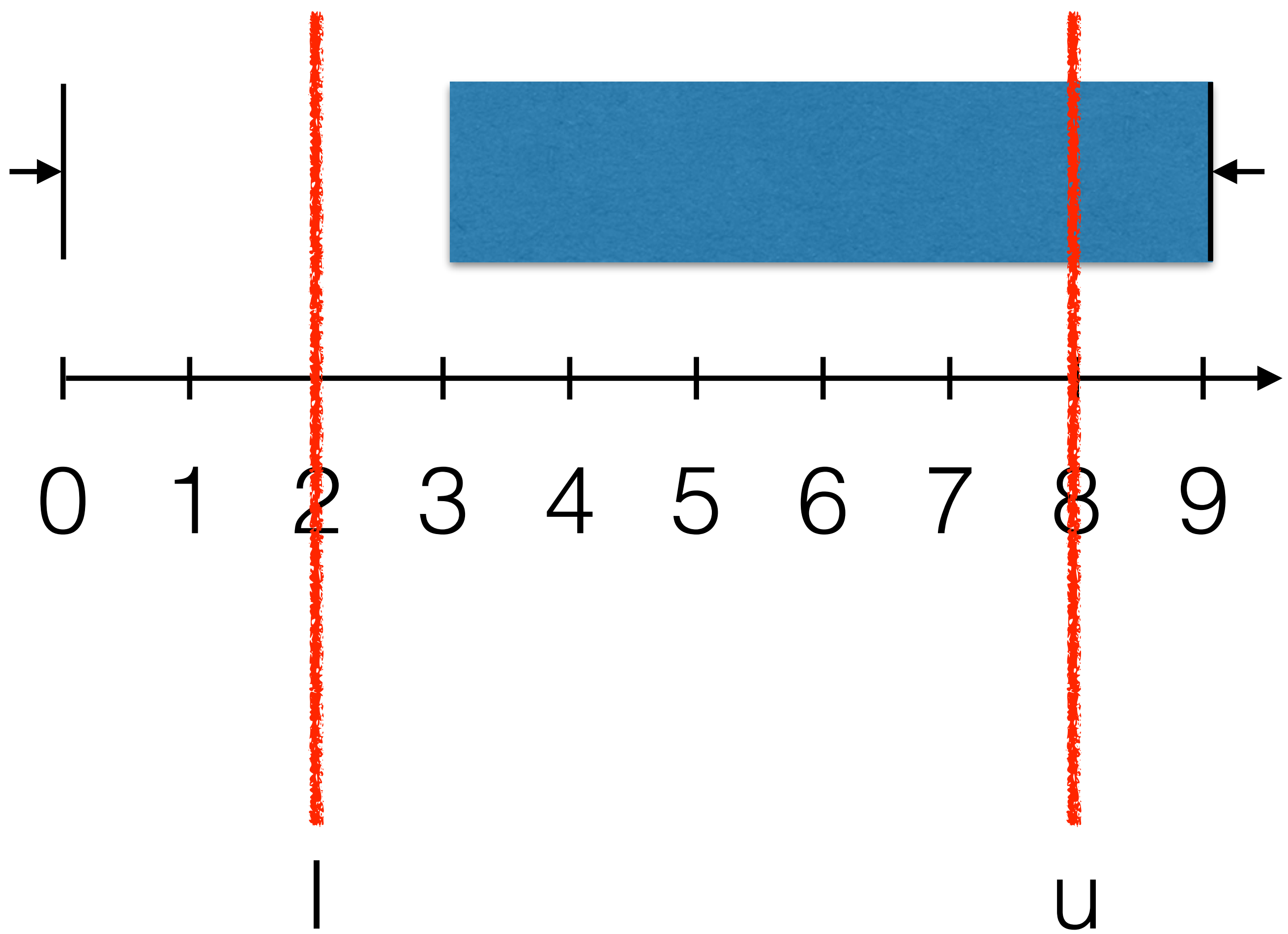

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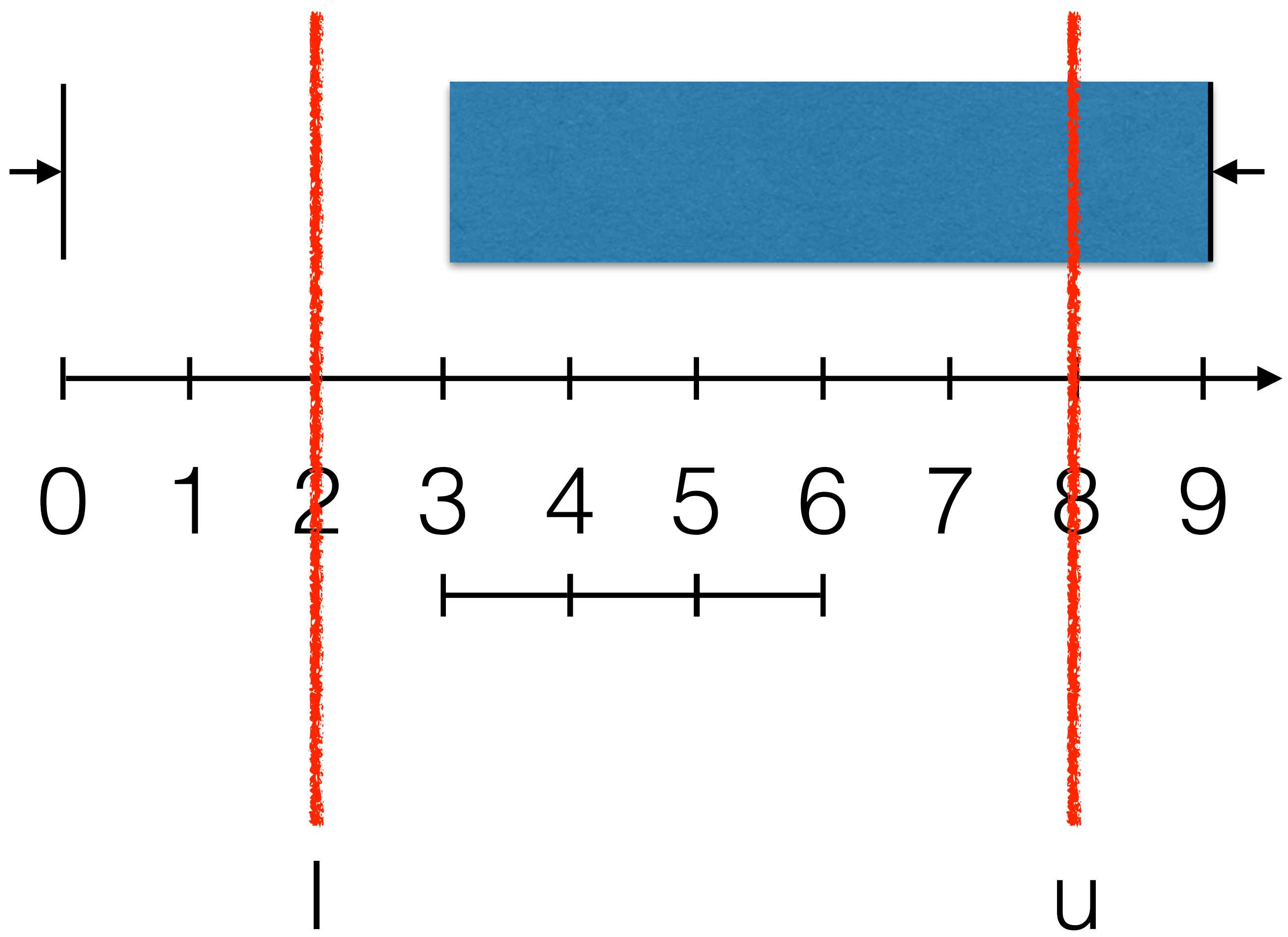

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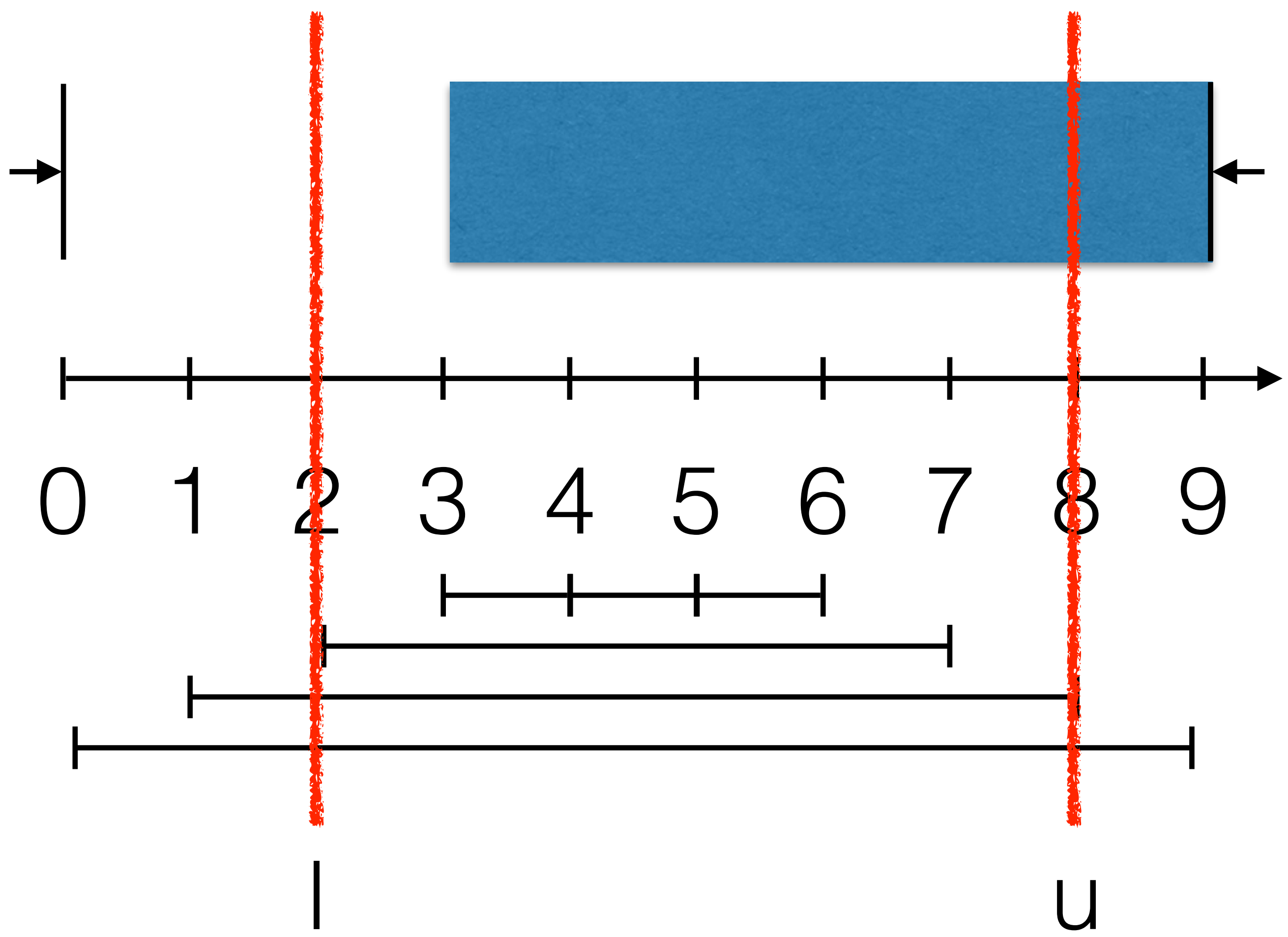

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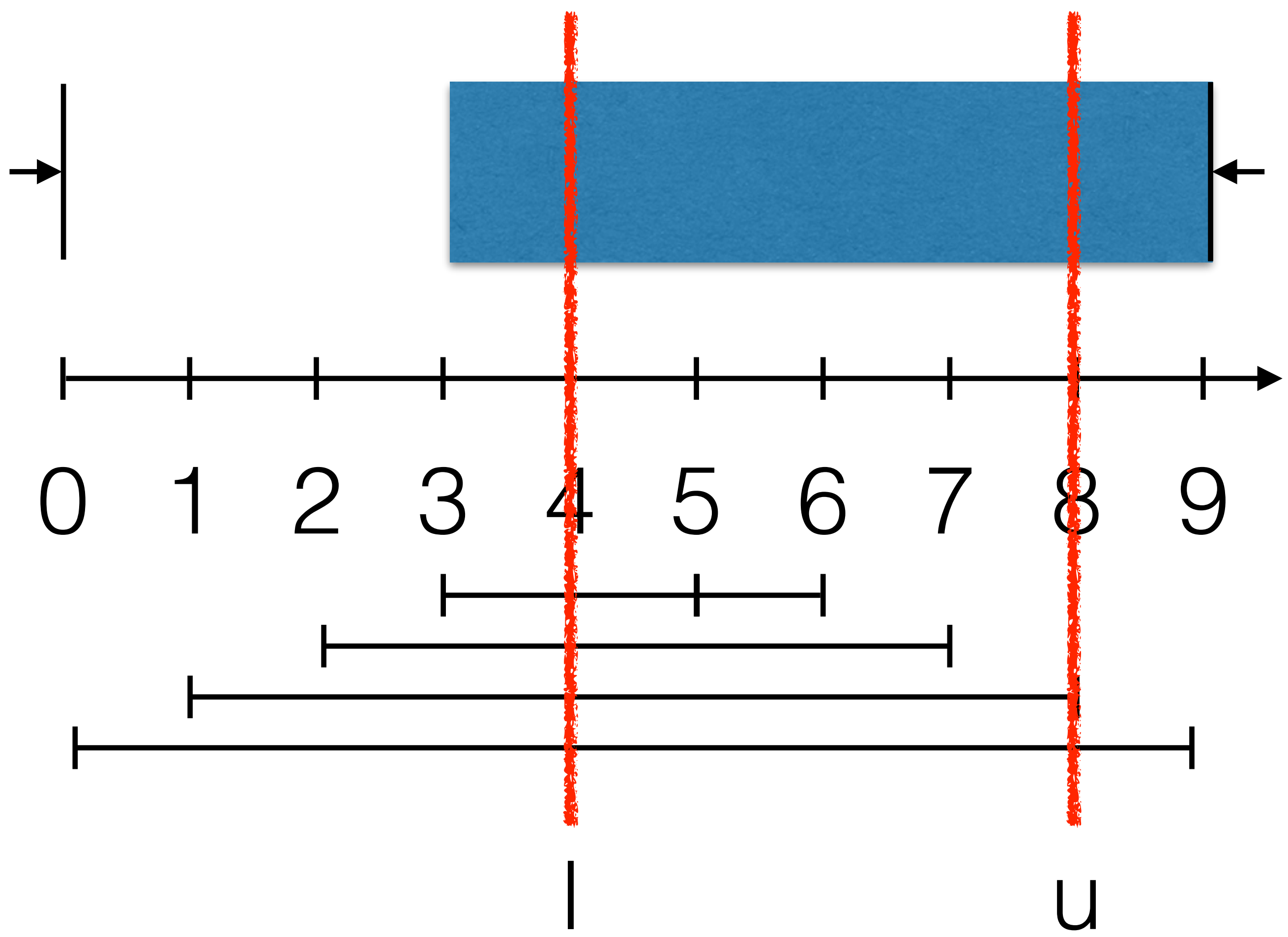

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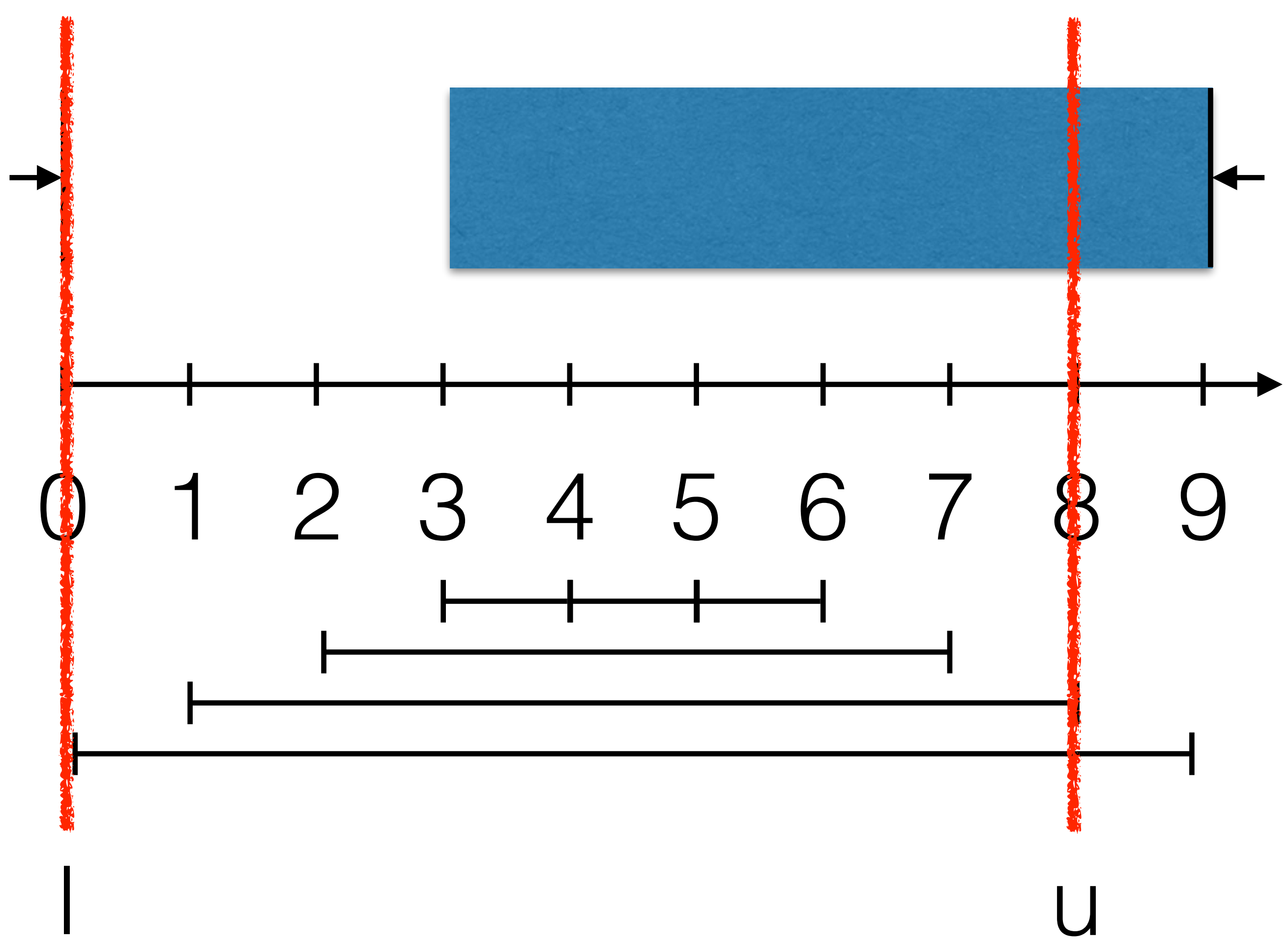

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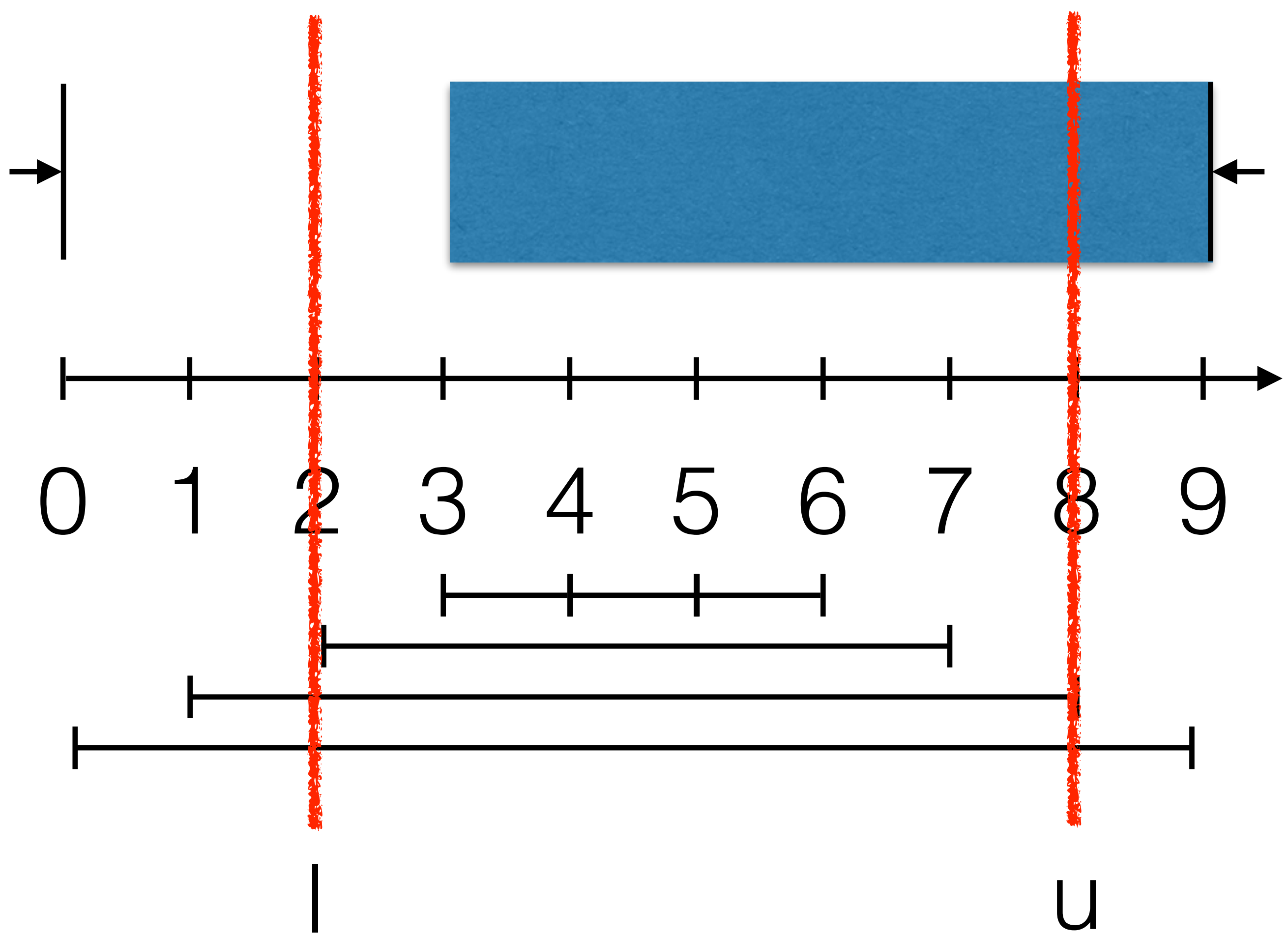

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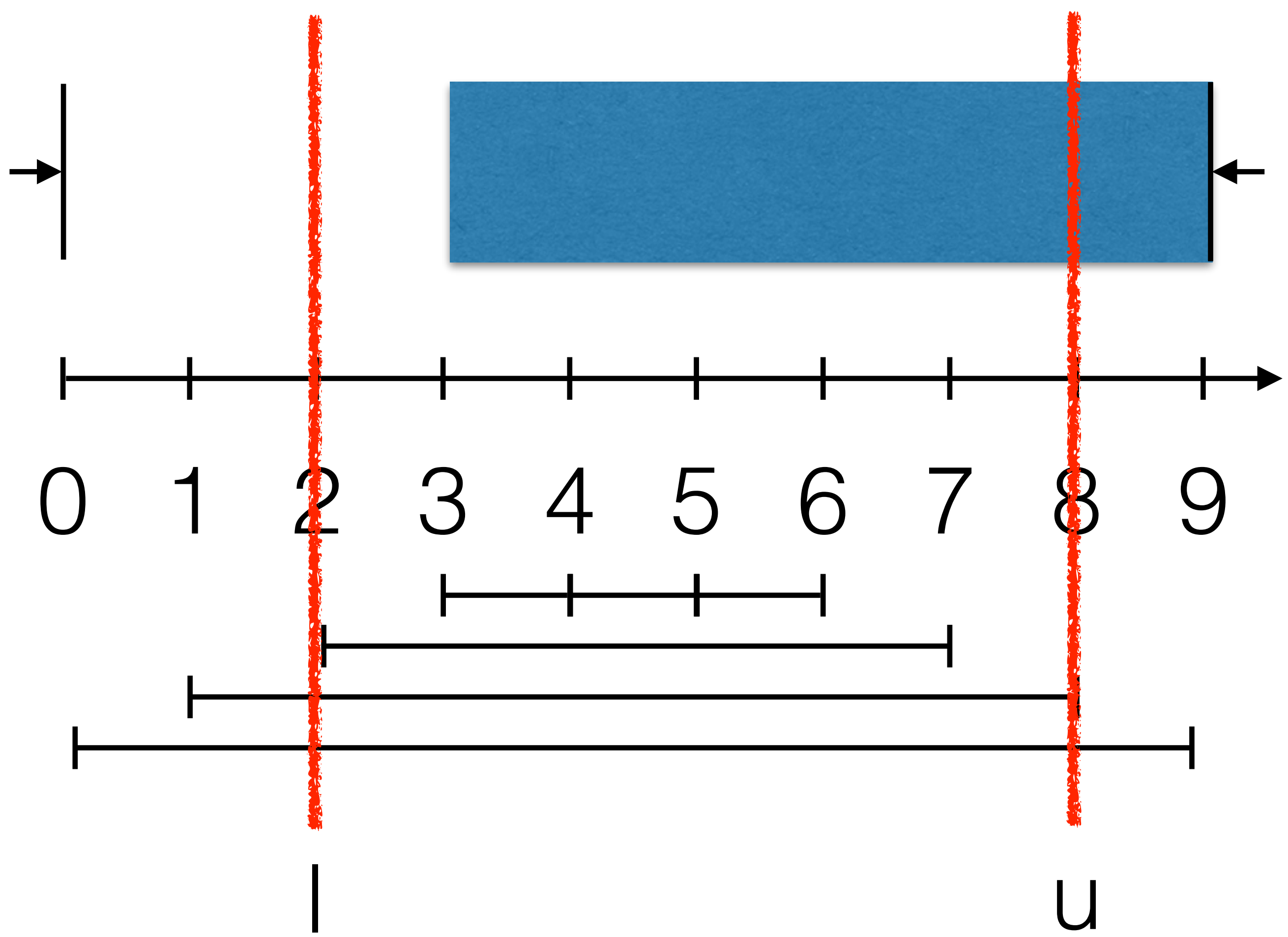

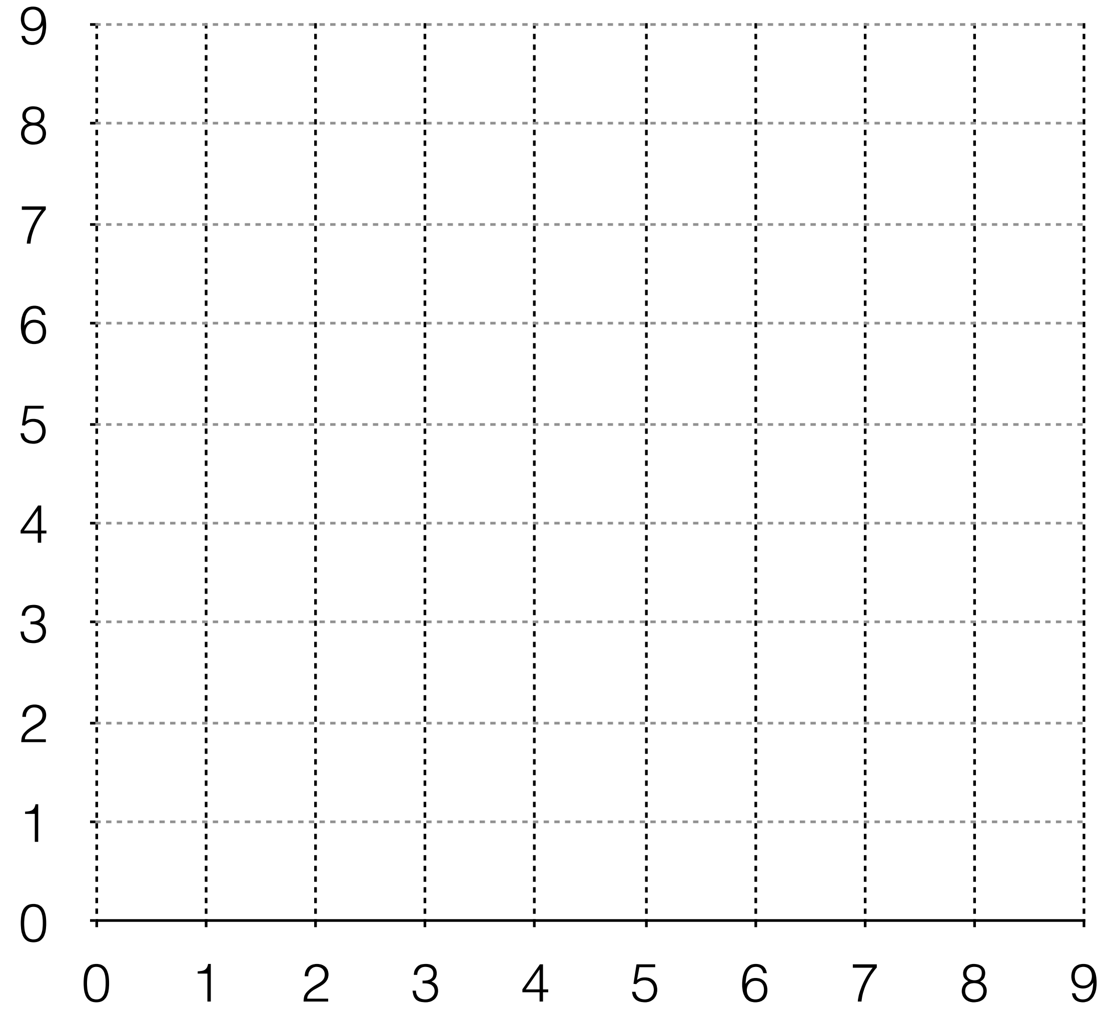

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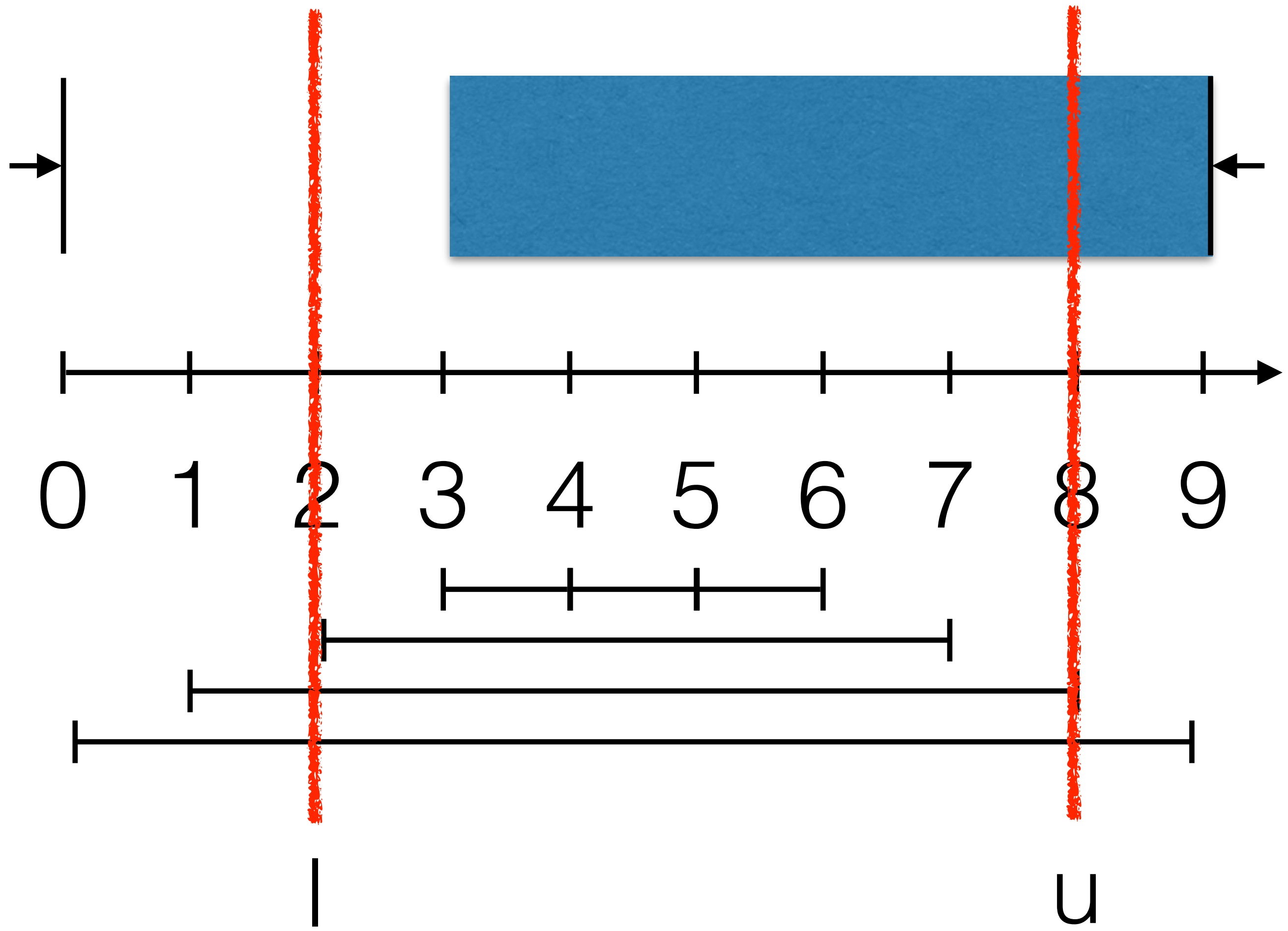

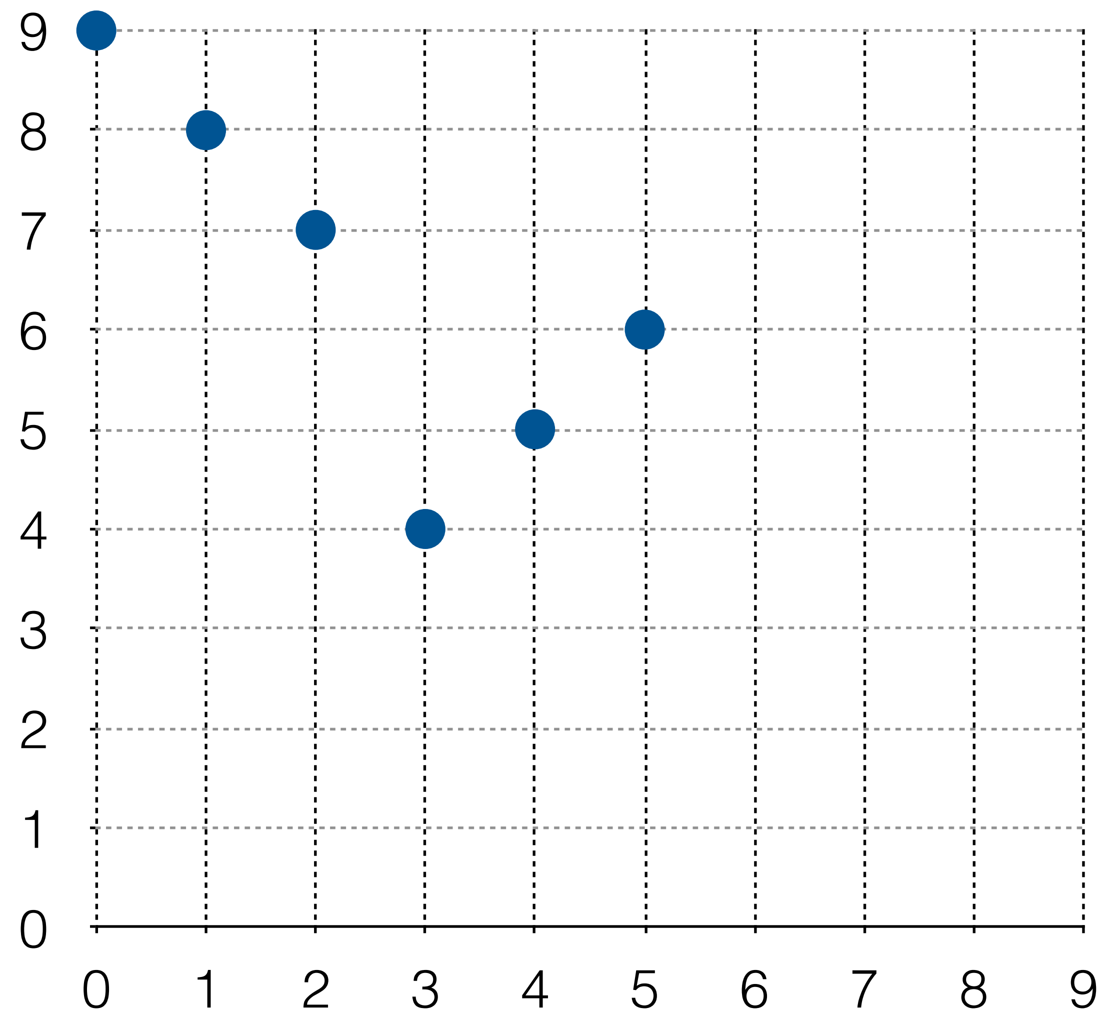

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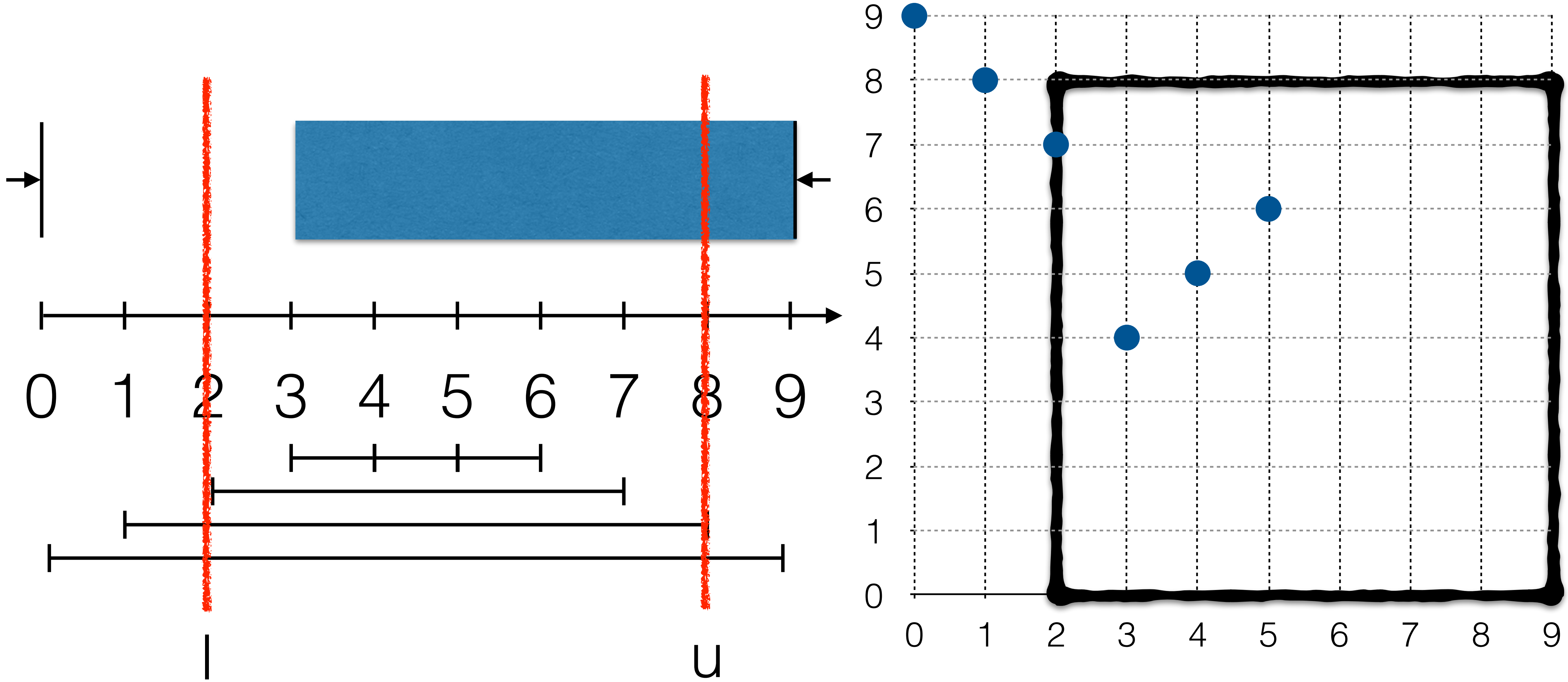

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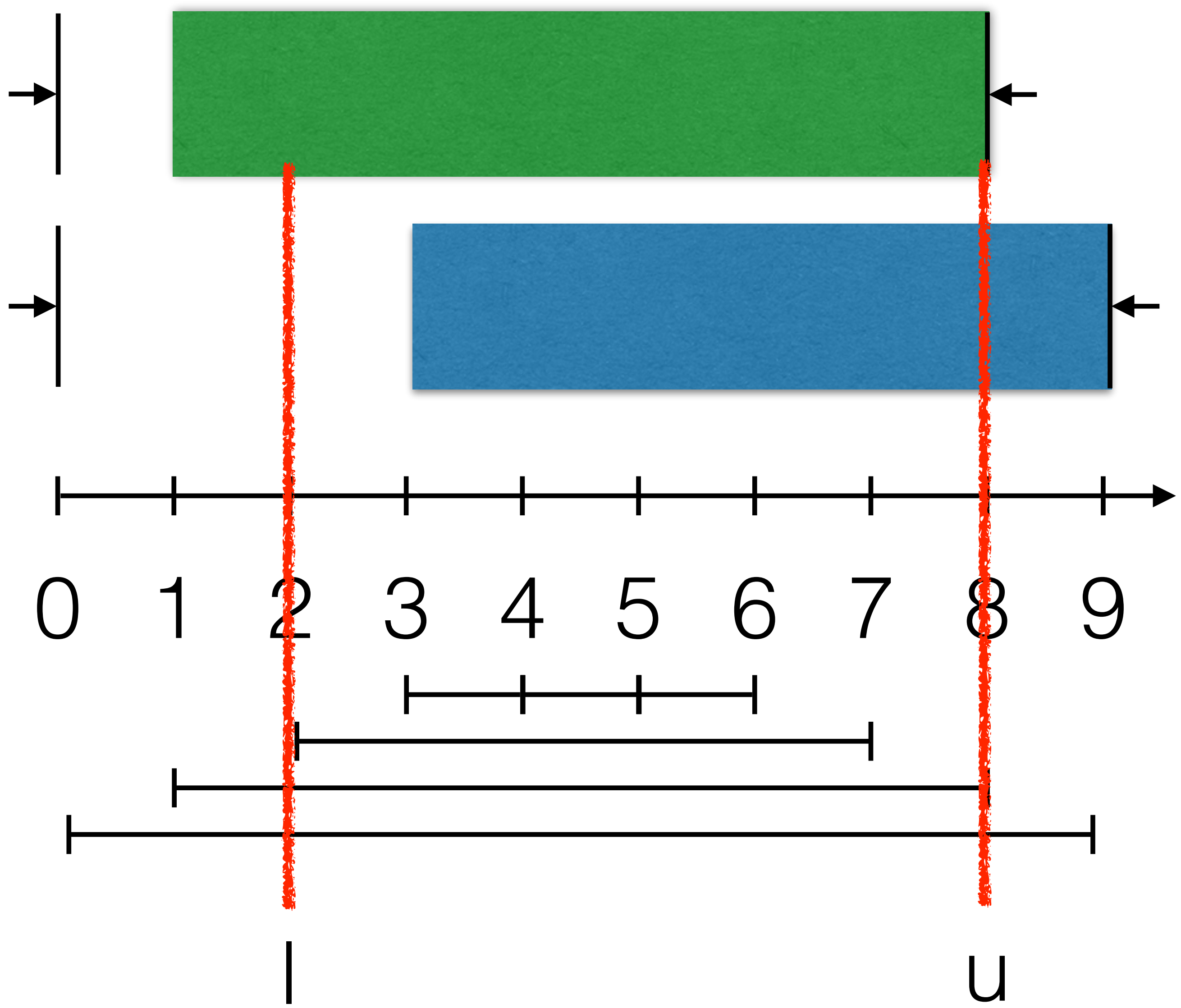

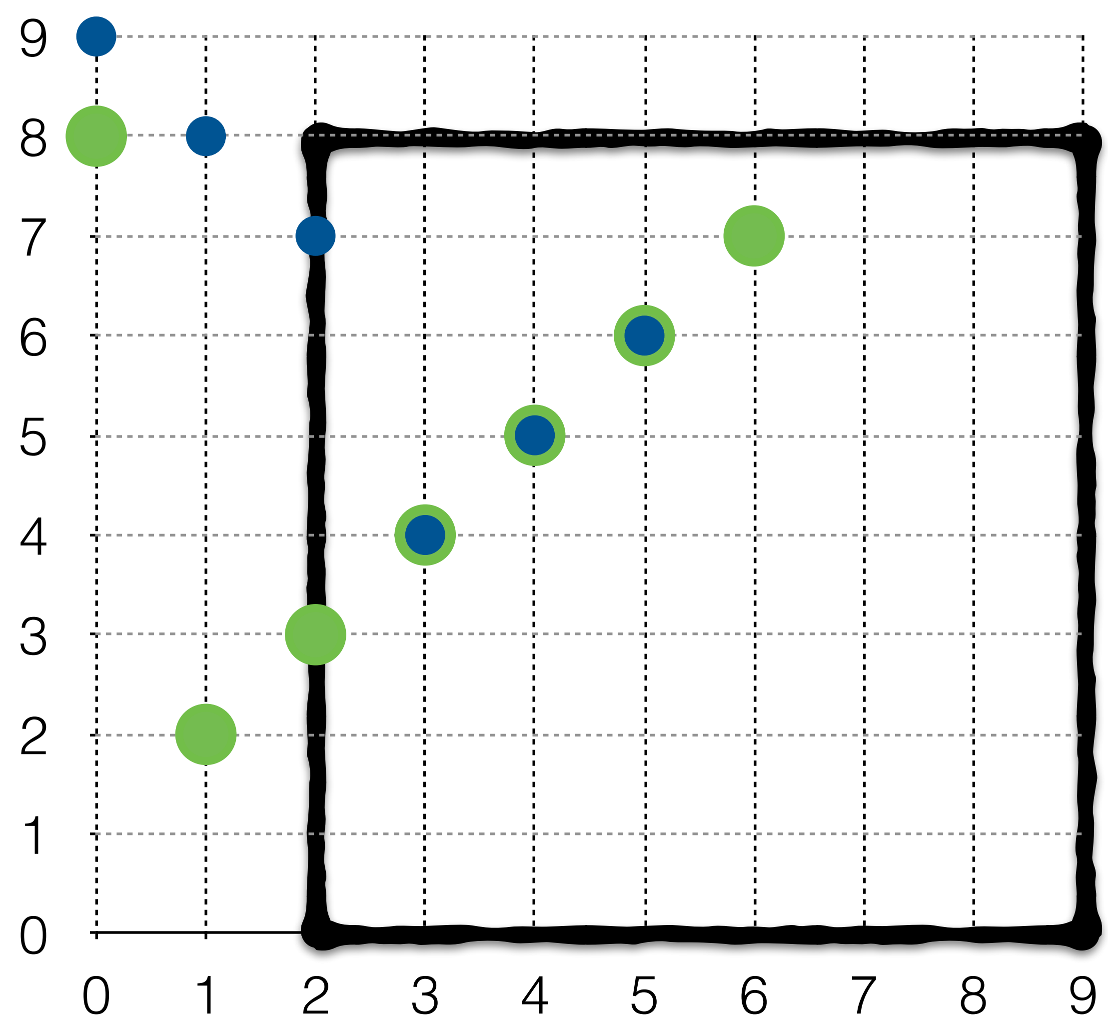

Range trees

- Given n points
- Build a range tree in $O(n \log n)$ space and time
- Count number of points in any given box in $O(\log n)$ time
- Since there are multiple points associated to a single task, adaptations are required to remain strongly polynomial in space and time.

How to check fewer than O(n^2) intervals?

- Recall that the slack is computed as follows.

$$S(l, u) = C \cdot (u - l) - \sum_i E(i, l, u)$$

- **Goal**: Find a time interval [*l*, *u*] such that *S*(*l*, *u*) < 0 while sampling fewer than O(n^2) intervals or guaranty that there is no such interval.

Advice #2

Increase your capacity to solve problems: learn new things every day.

How to improve your capacity to solve problems

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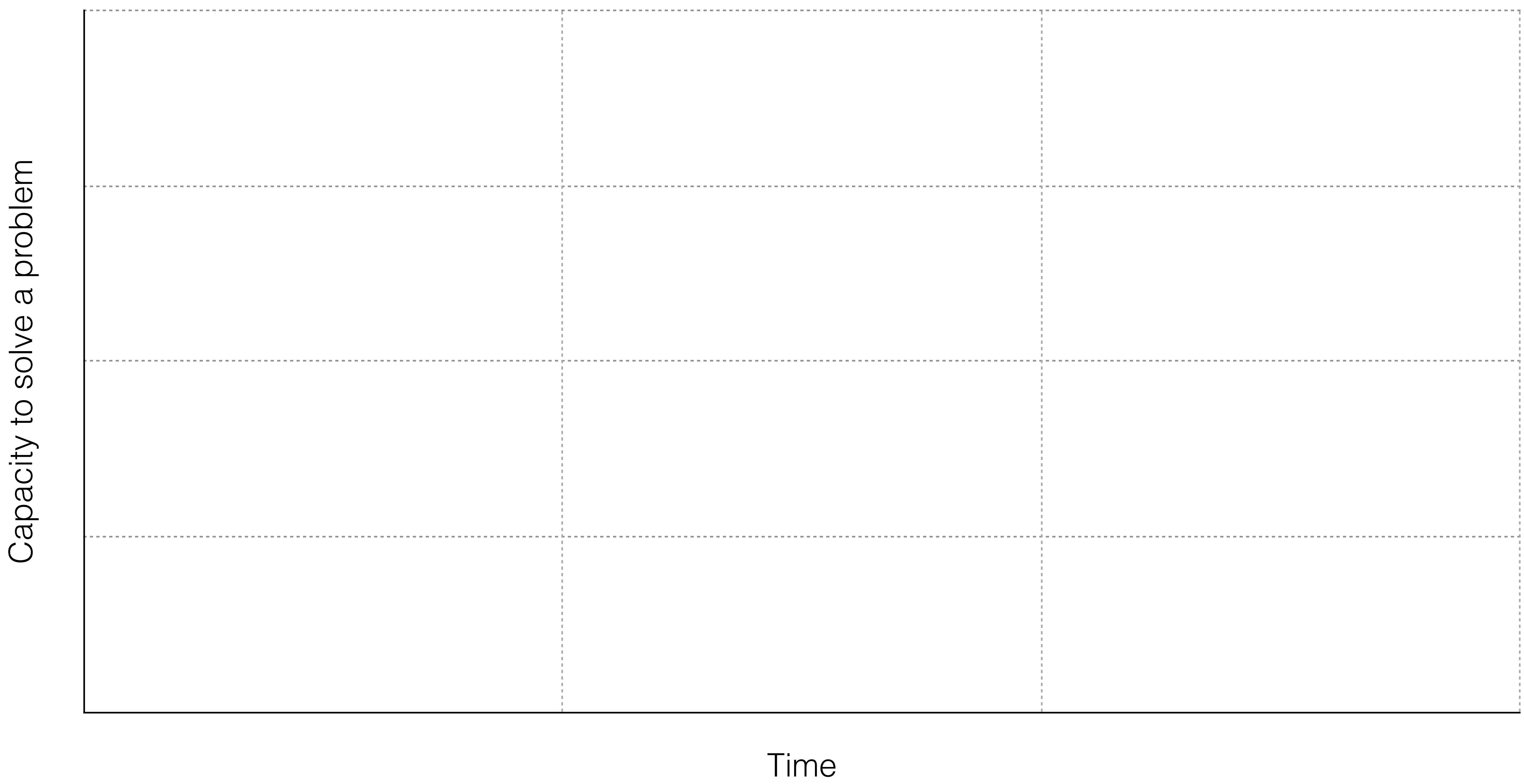

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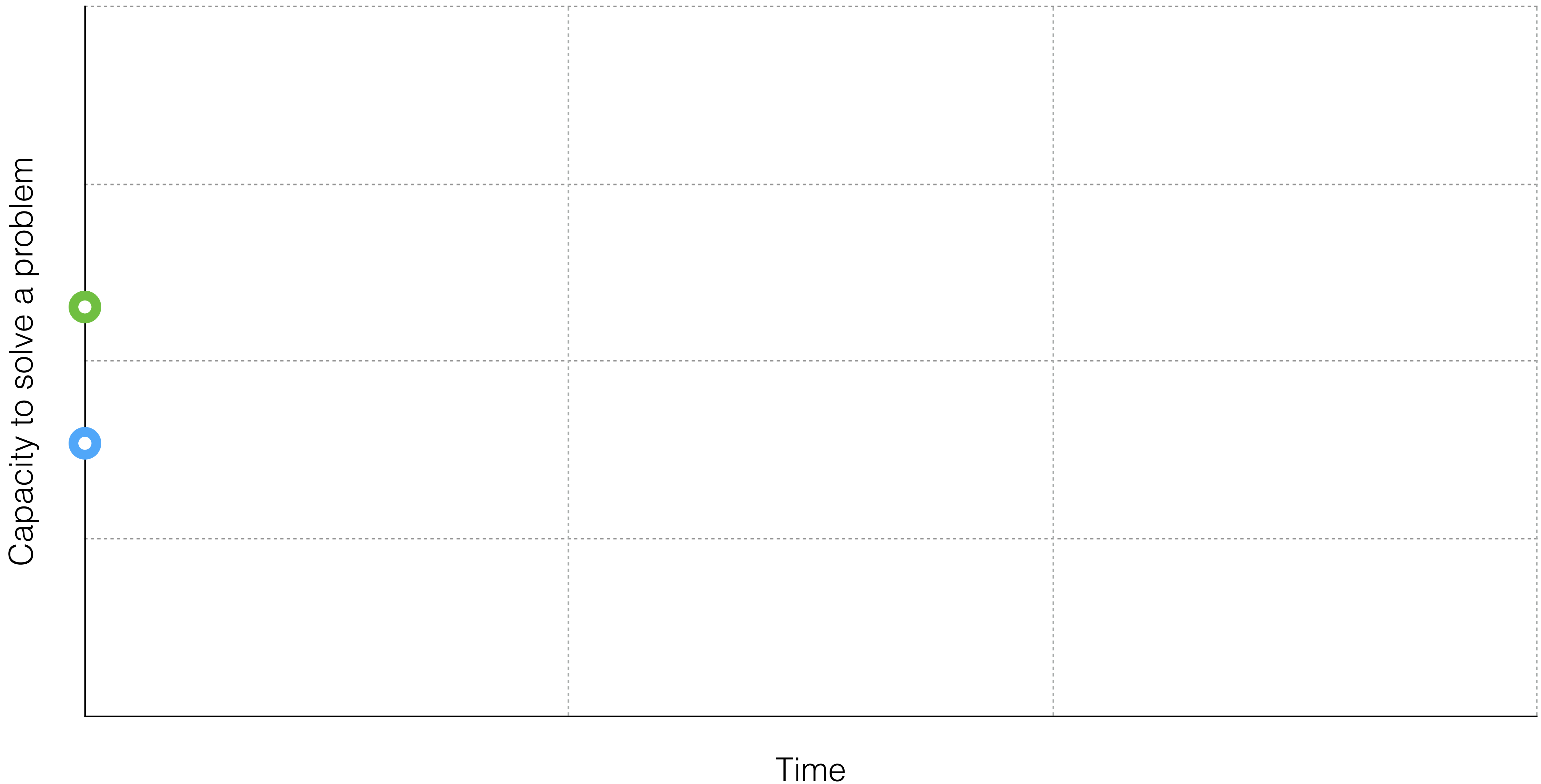

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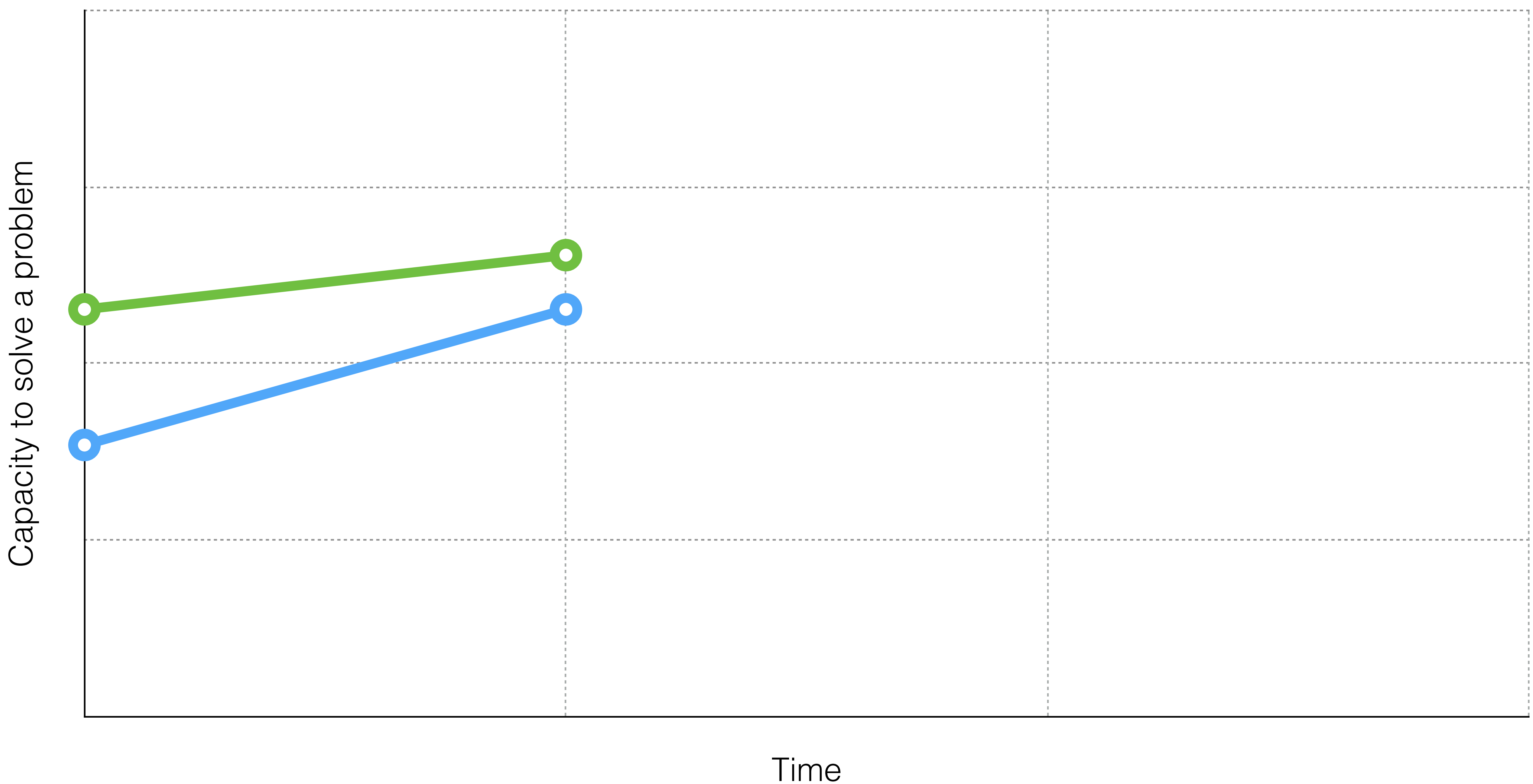

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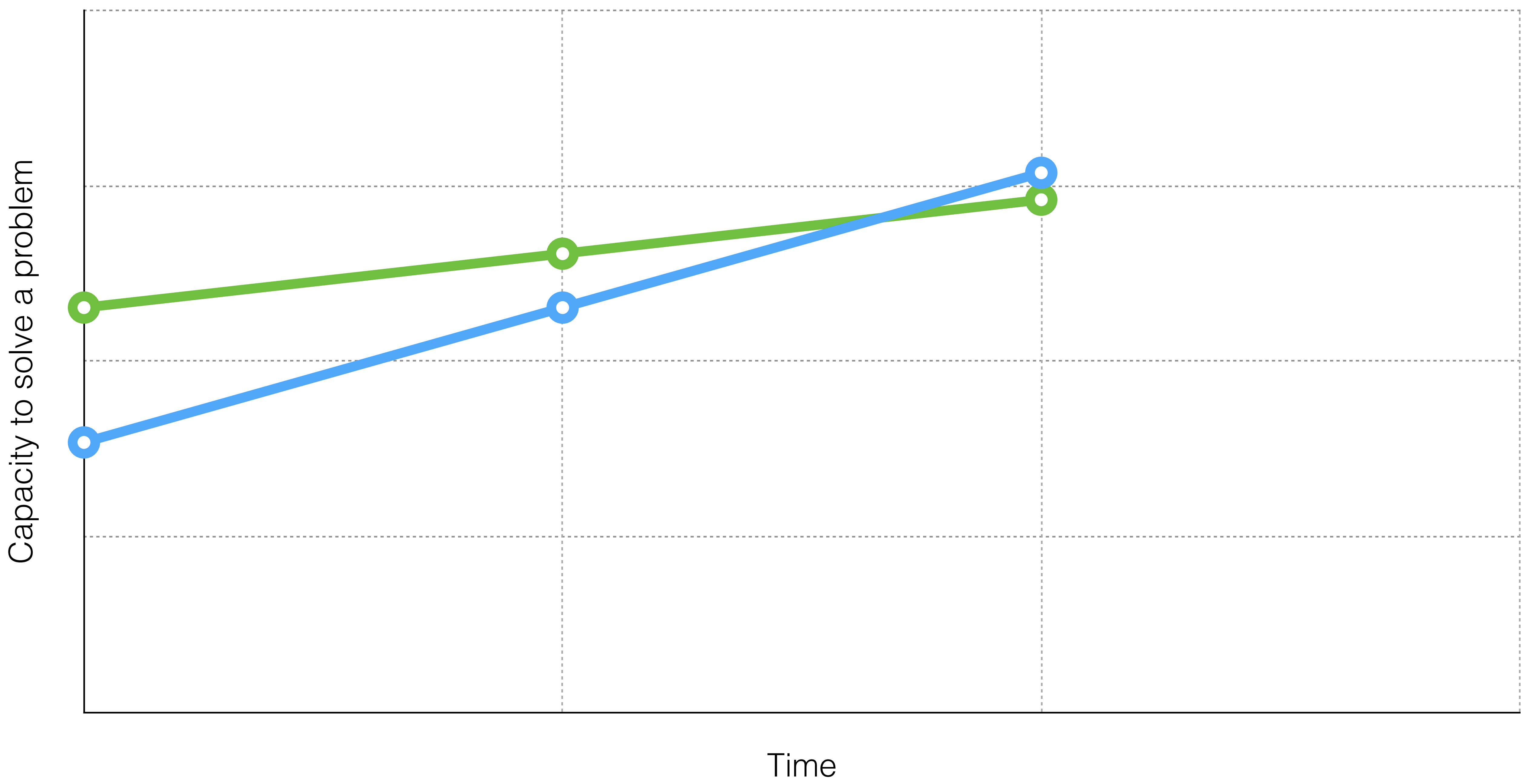

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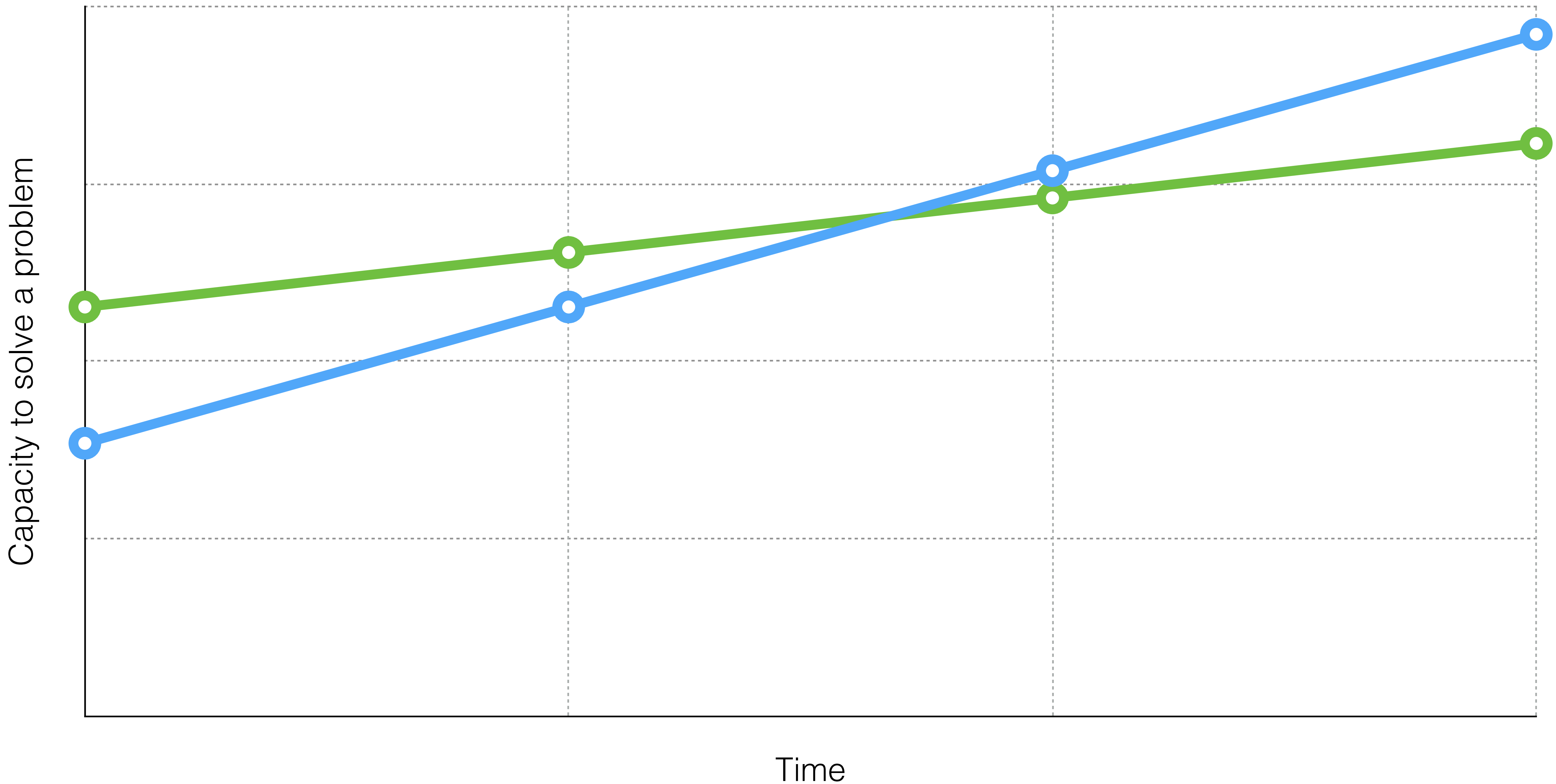

The slack matrix

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	2	4	6	8	10	8	8	7	6	6	8	10	12
1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
6							0	0	1	2	3	5	7	9
7								0	1	2	4	6	8	10
8									0	1	3	5	7	9
9										0	2	4	6	8
10											0	2	4	6
11												0	2	4
12													0	2
13														0

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1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
6							0	0	1	2	3	5	7	9
7								0	1	2	4	6	8	10
8									0	1	3	5	7	9
9										0	2	4	6	8
10											0	2	4	6
11												0	2	4
12													0	2
13														0

Monge matrix

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Inverse Monge matrix

$$M[i+1, j+1] - M[i+1, j] \geq M[i, j+1] - M[i, j]$$

- To understand the intuition of Monge matrices, consider the i^{th} row of a matrix as a function f_i.

$$\frac{f_{i+1}(x+1) - f_{i+1}(x)}{(x+1) - x} \geq \frac{f_i(x+1) - f_i(x)}{(x+1) - x}$$

- Function f_{i+1} grows faster than function f_i.
- Consequently, both functions cross each other only once.
- The crossing point (or region) can be computed with a binary search.

The slack matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	2	4	6	8	10	8	8	7	6	6	8	10	12
1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
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3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
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1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
6							0	0	1	2	3	5	7	9
7								0	1	2	4	6	8	10
8									0	1	3	5	7	9
9										0	2	4	6	8
10											0	2	4	6
11												0	2	4
12													0	2
13														0

The slack matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	2	4	6	8	10	8	8	7	6	6	8	10	12
1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
5						0	0	0	1	2	3	5	7	9
6							0	0	1	2	3	5	7	9
7								0	1	2	4	6	8	10
8									0	1	3	5	7	9
9										0	2	4	6	8
10											0	2	4	6
11												0	2	4
12													0	2
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2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
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The slack matrix

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	2	4	6	8	10	8	8	7	6	6	8	10	12
1		0	2	4	6	8	6	6	5	4	4	6	8	10
2			0	2	4	6	4	4	3	2	2	4	6	8
3				0	2	4	2	2	1	0	0	2	4	6
4					0	2	0	0	-1	0	1	3	5	7
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  - Overall complexity: $O(n \log^2 n)$
- But the dimension is not $n \times n$ …
- To obtain a complexity of $O(n \log^2 n)$, the algorithm only analyzes a subset of $O(n^2)$ cells characterized by Derrien and Petit.

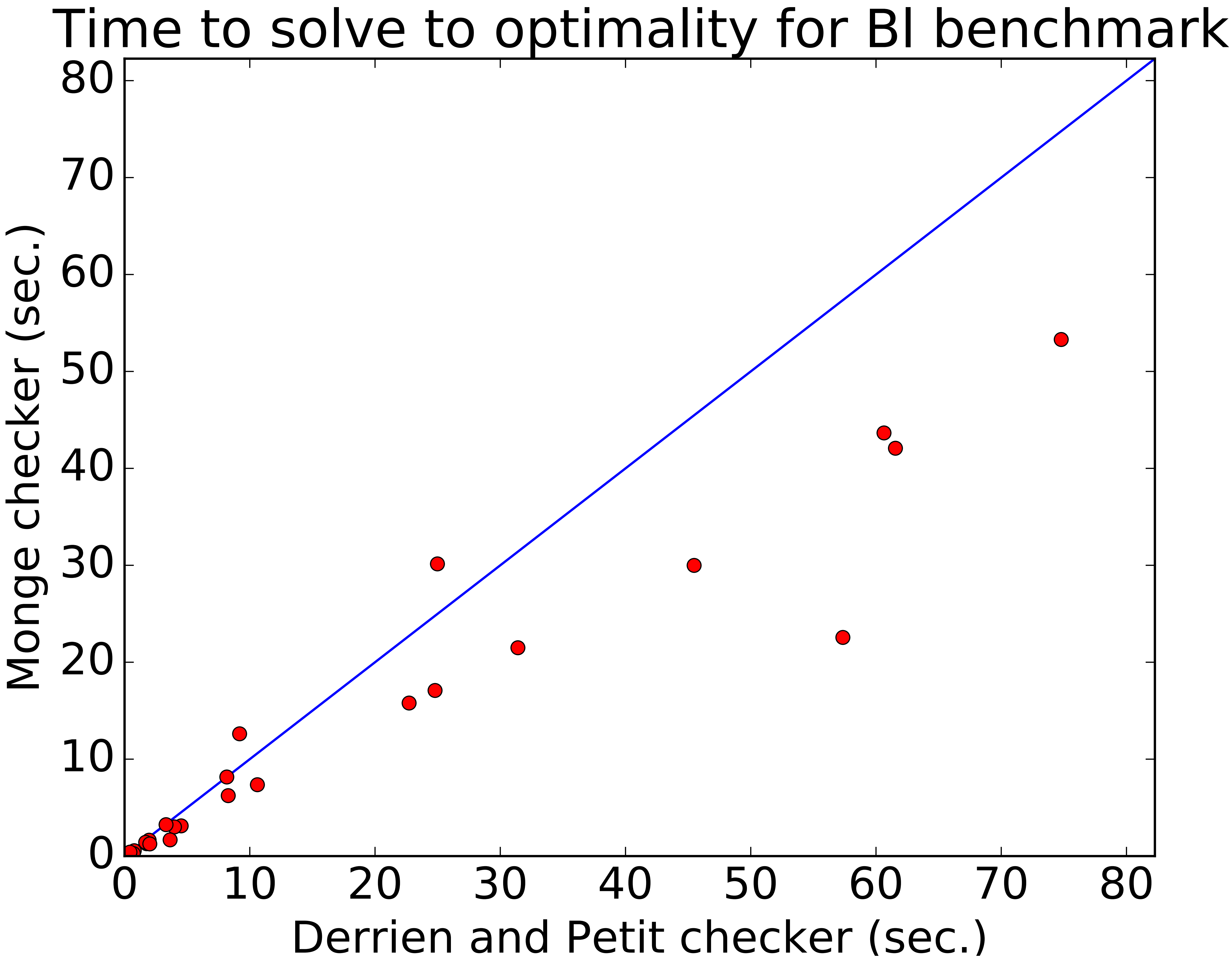
Advice #3

Make your research practical!

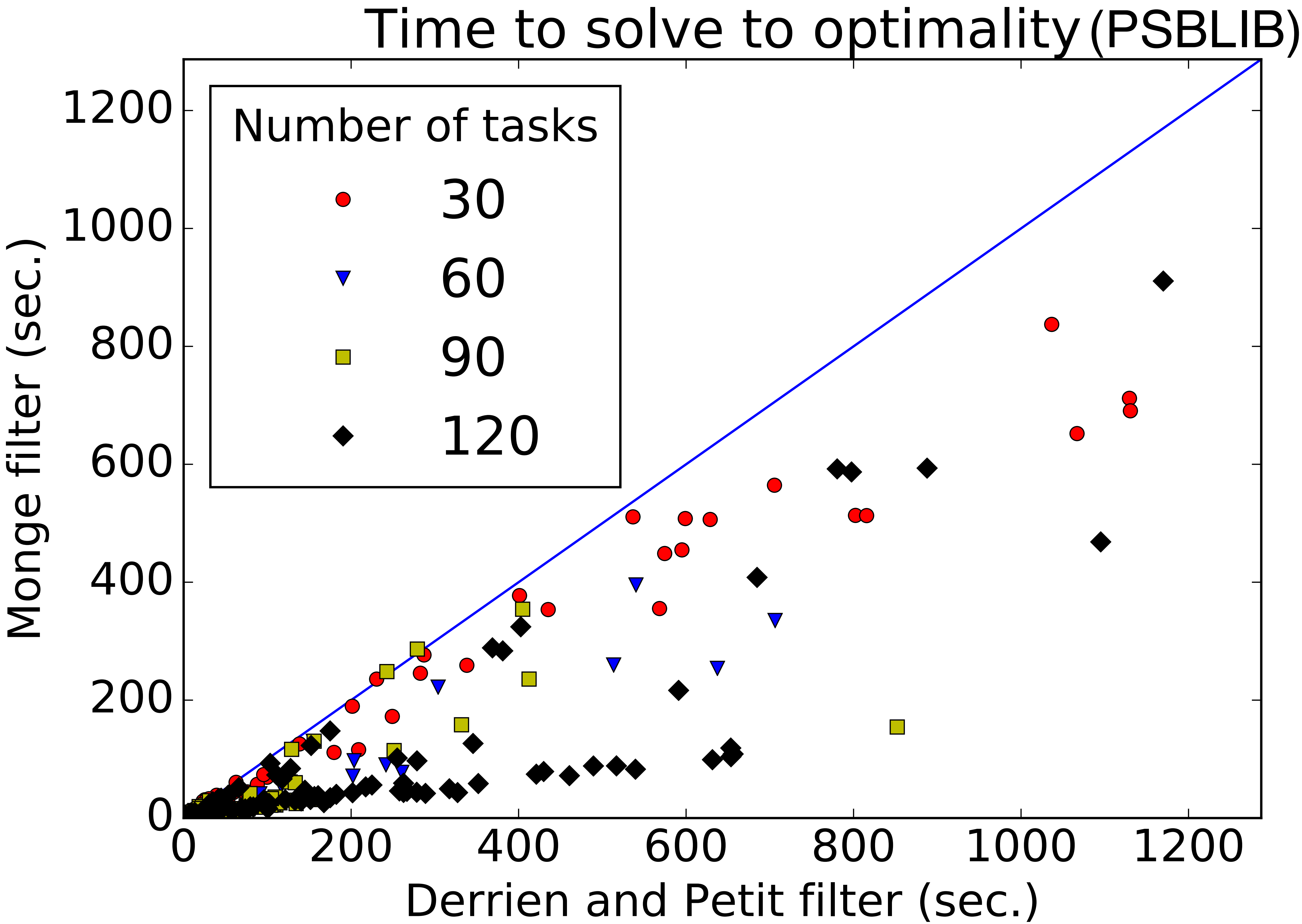
- If you want your graduate studies to leverage your industrial career, work on something practical.
- You prefer an academic career? Having industrial partners will help fund your lab.
- You prefer theory? No problem! Be prepared to justify with applications.
- Implement your ideas!

What we learned when implementing the algorithm

- A large portion of the computation is spent computing entries in the slack matrix.
    - Adding a cache prevents computing twice the same slack and save computation time.
- Derrien et Petit reduced the number of intervals of interests by a factor 7. This makes a huge difference for our checker as well.



Time to solve to optimality for BI benchmark
Monge checker (sec.)
Derrien and Petit checker (sec.)
0
10
20
30
40
50
60
70
80



Time to solve to optimality (PSBLIB)
Number of tasks
30
60
90
120
Monge filter (sec.)
Derrien and Petit filter (sec.)
0
200
400
600
800
1000
1200

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- Nogood learning

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Advice #4

Share your ideas

Conclusion

- Range trees are convenient to compute the amount of energy in a given time interval.
- The Monge property appears in scheduling problems and can be exploited.
- Energetic check in $O(n \log^2 n)$ time.

Advice #1

Reduce your problem to one that is already solved.

Advice #2

Increase your capacity to solve problems: learn new things every day.

Advice #3

Make it practical!

Advice #4

Share your ideas