From Linear Relaxations to Global Constraint Propagation[1]

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Recently, many algorithms have been designed to propagate global constraints. Unfortunately, some global constraints, such the AT-MOST-1 constraint and the EXTENDED-GCC are NP-Hard to propagate. Often, these constraints can easily be written as integer linear programs. Using linear relaxations and other techniques developed by the operation research community, we want to efficiently propagate such constraints.

We model constraints as integer programs that we relax into linear programs. For each value v in a variable domain dom(x), we create a binary variable x_v. The assignment $x_v = 1$ indicates that $x = v$ while $x_v = 0$ indicates that $x \neq v$. The binary variables are subject to the following linear program.

$$\left.\begin{array}{c} A\boldsymbol{x} \leq \boldsymbol{b} \\ 0 \leq x_i \leq 1 \end{array}\right\} \mathbf{P}$$

We find a solution to the relaxation using the interior point method. This method always converges to the interior of the solution polytope. Based on this observation, we conclude that if a variable x_i subject to $l \leq x_i \leq u$ is assigned to one of its boundary value l or u, the variable is assigned to this value in any solution. Therefore, if the interior point method assigns variable x_v to 0, we conclude that v does not have a support in dom(x).

We studied the consistency enforced by our propagator. We proved that if U is a totally unimodular matrix and that **P** has the following form, then GAC is enforced on the equations $U\boldsymbol{x} \leq \boldsymbol{b_1}$.

$$\left.\begin{array}{c} \begin{bmatrix} U & 0 \\ B & C \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} \leq \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} \\ 0 \leq x_i, y_j \leq 1 \end{array}\right\} \mathbf{P}$$

Some constraints like the CARDINALITY-MATRIX constraint while written in the form $A\boldsymbol{x} \leq \boldsymbol{b}$ have many totally unimodular sub-matrices of A. For each such submatrix, GAC is enforced on the corresponding variables. The resulting consistency is stronger than consistencies enforced by existing propagators.

References

1. O. Guler and Y. Ye *Convergence behavior of interior-point algorithms* Mathematical Programming 60, pages 215–228, 1993.

[1] This is joint work with Emmanuel Hebrard and Toby Walsh.