La planification d'horaires de ligues sportives par programmation par contraintes

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8 mars 2013

Objectifs de cette présentation

- Meilleure compréhension de la planification d'horaires de ligues sportives
- C'est un domaine complexe avec une notion de qualité d'horaire difficile à exprimer
- La programmation par contraintes est très appropriée pour résoudre ce problème

Outline

Sports Scheduling

- plan a tournament between several teams
  (set of games between pairs of teams)
- decide when (*round*) and where (*venue*) each game is played
- at a given game one team plays *home* and the other, *away*

A schedule may be . . .

- compact: each team plays at every round
- round robin: teams play each other a fixed nb of times
  single round robin (SRR); double round robin (DRR)
- mirrored: the second half of the schedule is a repetition of the first half, with the venues swapped

Traveling Tournament Problem (Major League Baseball)

TTP

- compact, double round robin schedule
- no more than three consecutive games at home or away
- minimize total distance traveled by all the teams

TTP with Predefined Venues (TTPPV)

- *single* round robin
- each pair of teams meet at predefined venue (SRR+SRR)
- instance is *balanced* if difference between number of home and away games is at most one for any given team

Example

TTPPV schedule for 6-team unbalanced instance

round	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
team 1:	@2	@3	6	@5	@4
team 2:	1	6	@5	@4	@3
team 3:	5	1	4	@6	2
team 4:	@6	@5	@3	2	1
team 5:	@3	4	2	1	@6
team 6:	4	@2	@1	3	5

- compact, single round robin
- no more than three consecutive games at home or away
- each pair of teams meet at predefined venue
- unbalanced instance

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Constraint Programming (CP)

- problem described through formal model expressed using constraints from rich set of modeling primitives
- variables take their value from a finite set (*domain*)
- each type of constraint encapsulates its own specialized inference algorithm
- sophisticated inference to reduce search space
- combination of variable- and value-selection heuristics to guide exploration of search space

CP for Sports Scheduling

- instrumental in scheduling Major League Baseball in North America
- used in the past to schedule the National Football League in the US
- also applied to College Basketball

Back to the TTPPV...

Decision Variables

- n teams (and hence $n-1$ rounds)
- o_{ij}, opponent of team i in round j
- $h_{ij} = 1$ iff team i plays at home in round j

round	1^{st}	...	j^{th}	...	$(n-1)^{th}$
team 1:					
⋮					
team i:			$h_{ij}; o_{ij}$		
⋮					
team n:					

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- o_{ij}, opponent of team i in round j
- $h_{ij} = 1$ iff team i plays at home in round j

round	1^{st}	...	j^{th}	...	$(n-1)^{th}$
team 1:					
⋮					
team i:			$0; k$		
⋮					
team n:					

Decision Variables

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- o_{ij}, opponent of team i in round j
- $h_{ij} = 1$ iff team i plays at home in round j

round	1^{st}	...	j^{th}	...	$(n-1)^{th}$
team 1:					
⋮					
team i:			@k		
⋮					
team n:					

Basic Structure

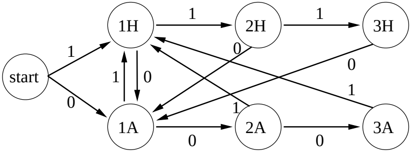
$$\begin{aligned} o_{ij} &\in \{1, \ldots, i-1, i+1, \ldots, n\} && 1 \leq i \leq n, 1 \leq j \leq n-1 \\ h_{ij} &\in \{0, 1\} && 1 \leq i \leq n, 1 \leq j \leq n-1 \\ \texttt{alldifferent}((o_{ij})_{1 \leq j \leq n-1}) & && 1 \leq i \leq n \\ o_{o_{ij}, j} &= i && 1 \leq i \leq n, 1 \leq j \leq n-1 \end{aligned}$$

Predefined Venues

$$
\begin{aligned}
o_{ij} &\in \{1, \ldots, i-1, i+1, \ldots, n\} && 1 \le i \le n, 1 \le j \le n-1 \\
h_{ij} &\in \{0, 1\} && 1 \le i \le n, 1 \le j \le n-1 \\
\texttt{alldifferent}((o_{ij})_{1 \le j \le n-1}) & && 1 \le i \le n \\
o_{o_{ij}, j} &= i && 1 \le i \le n, 1 \le j \le n-1 \\
h_{ij} &= V[i, o_{ij}] && 1 \le i \le n, 1 \le j \le n-1
\end{aligned}
$$

At most 3 consecutive home or away games

$$
\begin{array}{rl}
o_{ij} \in \{1,\ldots,i-1,i+1,\ldots,n\} & 1 \le i \le n, 1 \le j \le n-1 \\
h_{ij} \in \{0,1\} & 1 \le i \le n, 1 \le j \le n-1 \\
\texttt{alldifferent}((o_{ij})_{1 \le j \le n-1}) & 1 \le i \le n \\
o_{o_{ij},j} = i & 1 \le i \le n, 1 \le j \le n-1 \\
h_{ij} = V[i, o_{ij}] & 1 \le i \le n, 1 \le j \le n-1 \\
\texttt{regular}((h_{ij})_{1 \le j \le n-1}, \mathcal{A}) & 1 \le i \le n
\end{array}
$$



Adding travel costs

- D : $n \times n$ travel distance matrix
- vars d_{ij} : travel distance for team i to go play in round j

round	1^{st}	...	j^{th}	...	$(n-1)^{th}$
team 1:					
⋮					
team i:			$h_{ij}; o_{ij}; d_{ij}$		
⋮					
team n:					

Adding travel costs

- D : $n \times n$ travel distance matrix
- vars d_{ij} : travel distance for team i to go play in round j

Individual travel distances depend on pairs of consecutive variables:

$$(h_{ij} = 1 \wedge h_{i,j+1} = 1) \Rightarrow d_{i,j+1} = 0 \qquad 1 \leq i \leq n, 1 \leq j < n\text{-}1$$

$$(h_{ij} = 1 \wedge h_{i,j+1} = 0) \Rightarrow d_{i,j+1} = D[i, o_{i,j+1}] \qquad 1 \leq i \leq n, 1 \leq j < n\text{-}1$$

$$(h_{ij} = 0 \wedge h_{i,j+1} = 1) \Rightarrow d_{i,j+1} = D[o_{ij}, i] \qquad 1 \leq i \leq n, 1 \leq j < n\text{-}1$$

$$(h_{ij} = 0 \wedge h_{i,j+1} = 0) \Rightarrow d_{i,j+1} = D[o_{ij}, o_{i,j+1}] \qquad 1 \leq i \leq n, 1 \leq j < n\text{-}1$$

$$d_{ij} \in \{0\} \cup \{\min\{D\}, \ldots, \max\{D\}\} \qquad 1 \leq i \leq n, 1 \leq j \leq n$$

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}$$

At this point we have a complete model

Adding redundant constraints

In a round j, each team appears exactly once as an opponent:

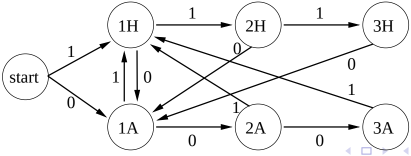
$$\texttt{alldifferent}((o_{ij})_{1 \le i \le n}) \qquad 1 \le j \le n-1$$

Catching infeasible instances

- Need enough home (resp. away) games to separate long stretches of away (resp. home) games
- Necessary condition: at least $\lfloor \frac{n-1}{4} \rfloor$ of each are needed (Melo,Urrutia,Ribeiro, *J. Scheduling*, 2009)

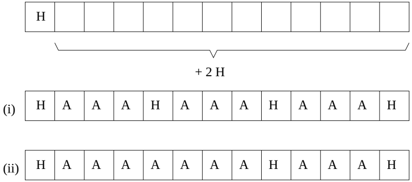
Combine "3-home/3-away" constraint with counting number of home games:

$$\texttt{cost\_regular}((h_{ij})_{1 \le j \le n-1},\ \mathcal{A},\ \textstyle\sum_{1 \le k \le n} V[i,k],\ [0,1]) \qquad 1 \le i \le n$$



Catching infeasible subinstances

- Consider 14-team individual schedule with 3 predefined home games
- Necessary condition satisfied: $\lfloor \frac{14-1}{4} \rfloor = 3$
- New constraint more powerful because used dynamically:



Both (i) and (ii) completed schedules are invalid

Symmetry breaking

Forbid the mirror image of a schedule, going from the last round to the first (travel distances are symmetric):

$$o_{11} < o_{1,n-1}$$

Some generic variable-selection heuristics

Branching on opponent variables

- *lexico*: Select variables in lexicographic order (row by row)
- *random*: Select variables randomly
- *dom*: Select the variable with the fewest alternatives

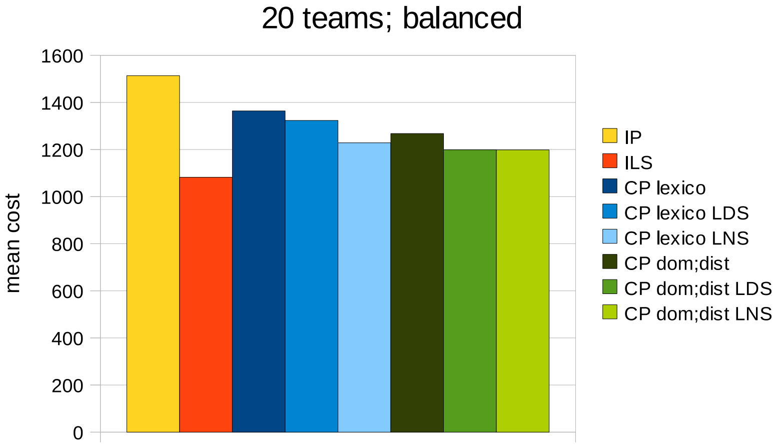
Dedicated value-selection heuristic

At this point, we have selected one team and the round;
value selection determines the other team (and thus a game)

- *dist*: For that game (in both rows), compute lower bound on travel distances from previous round and to next round. Select the value with the smallest lower bound.

Benchmark Instances

- Existing TTP instances (Circle instances) to which a venue is added for each game (Melo,Urrutia,Ribeiro, *J. Scheduling*, 2009)
- Up to 20 teams; twenty instances (ten balanced; ten nonbalanced) of each size


20 teams; balanced
mean cost
1600
1400
1200
1000
800
600
400
200
0
IP
ILS
CP lexico
CP lexico LDS
CP lexico LNS
CP dom;dist
CP dom;dist LDS
CP dom;dist LNS

Brazilian Football League

- compact, mirrored double round robin schedule
- 20 teams
- *many* restrictions
- maximize gate attendance and TV audience

Decision Variables

- $n = 20$ teams (and hence 38 rounds)
- o_{ij} : opponent of team i in round j
- $h_{ij} = 1$ iff team i plays at home in round j
- r_{ik} : first-half round in which team i plays against k
- $h'_{ik} = 1$ iff team i plays at home against k in first half
- b_{ij} : away break (0), home break (2), or no break (1) for team i in round j

Basic Structure

$$
\begin{array}{rl}
o_{ij} \in \{1, \ldots, i-1, i+1, \ldots, n\} & 1 \leq i \leq n, 1 \leq j \leq 2(n-1) \\
h_{ij} \in \{0, 1\} & 1 \leq i \leq n, 1 \leq j \leq 2(n-1) \\
o_{o_{ij},j} = i & 1 \leq i \leq n, 1 \leq j \leq n-1 \\
h_{o_{ij},j} \neq h_{ij} & 1 \leq i \leq n, 1 \leq j \leq n-1 \\
\texttt{alldifferent}((o_{ij})_{1 \leq j \leq n-1}) & 1 \leq i \leq n \\
\texttt{alldifferent}((o_{ij})_{1 \leq i \leq n}) & 1 \leq j \leq n-1
\end{array}
$$

Basic Structure (continued)

$$
\begin{array}{rl}
r_{ik} \in \{0, \ldots, n-1\} & 1 \le i \le n, 1 \le k \le n \\
h'_{ik} \in \{0, 1\} & 1 \le i \le n, 1 \le k \le n \\
b_{ij} \in \{0, 1, 2\} & 1 \le i \le n-1, 1 \le j \le n-1 \\
r_{ii} = 0 & 1 \le i \le n \\
\texttt{alldifferent}((r_{ik})_{1 \le k \le n}) & 1 \le i \le n \\
r_{ik} = r_{ki} & 1 \le i \le n, 1 \le k \le n \\
h'_{ik} \ne h'_{ki} & 1 \le i \le n, 1 \le k \le n \\
h'_{ik} = h_{ir_{ik}} & 1 \le i \le n, i < k \le n \\
o_{ir_{ik}} = k & 1 \le i \le n, i < k \le n \\
b_{ij} = h_{ij} + h_{i,j+1} & 1 \le i \le n-1, 1 \le j \le n-1
\end{array}
$$

Basic Structure (continued)

$$
\begin{array}{rl}
r_{ik} \in \{0,\ldots,n-1\} & 1 \le i \le n, 1 \le k \le n \\
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h'_{ik} = h_{i r_{ik}} & 1 \le i \le n, i < k \le n \\
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b_{ij} = h_{ij} + h_{i,j+1} & 1 \le i \le n-1, 1 \le j \le n-1
\end{array}
$$

Mirrored Double Round Robin Structure

$$o_{i,(n-1)+j} = o_{ij} \qquad 1 \le i \le n, 1 \le j \le n-1$$
$$h_{i,(n-1)+j} \neq h_{ij} \qquad 1 \le i \le n, 1 \le j \le n-1$$

Particular groupings

Special types of games

- G12: games between the 12 best teams
- Classic: games between teams of the same city
- Regional: games between teams of the same state

Paired teams

- teams are paired: 1 with 2, 3 with 4, ...
- usually teams from the same city

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Special types of games

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Public Safety

Opposite home/away pattern for paired teams

$$h_{ij} \neq h_{i+1,j} \qquad i \in \{1,3,5,\ldots,n-1\}, 1 \leq j \leq n-1$$

At most 1 Classic game per city per round

$$\texttt{alldifferent}((r_{ik})_{(i,k)\in \text{Classic}_j}) \qquad j \in \text{“>2 team cities”}$$

Revenues

Maximize classic games on double-weekend rounds

$$\sum_{1 \leq i \leq n,\ j \in DW} ((i, o_{ij}) \in \text{Classic}) \geq 2 \times LB$$

Number of G12 games in each round is between 2 and 4

$$\text{gcc}((r_{ik})_{(i,k) \in G12}, \langle g_1, \ldots, g_{n-1} \rangle)$$

$$2 \leq g_j \leq 4 \qquad 1 \leq j \leq n-1$$

Attractiveness

No regional (or classic) games in first 3 rounds

$$r_{ik} \notin \{1,2,3\} \qquad (i,k) \in \text{Regional}$$

No G12 RdJ-SP game in same round as Classic RdJ or SP game

$$r_{ik} \neq r_{i'k'} \qquad (i,k) \in \text{Classic RdJ or SP}, (i',k') \in \text{G12 RdJ-SP}$$

Fairness

First team of a pair plays home in first round and away in last round

$$h_{i1} = 1 \wedge h_{i,n-1} = 0 \qquad i \in \{1, 3, 5, \ldots, n-1\}$$

Fairness

No breaks in first 4 or last 2 rounds

$$
\begin{aligned}
b_{i1} &= 1 & 1 \leq i \leq n \\
b_{i2} &= 1 & 1 \leq i \leq n \\
b_{i3} &= 1 & 1 \leq i \leq n \\
b_{i,n-2} &= 1 & 1 \leq i \leq n
\end{aligned}
$$

No regional games in last 4 rounds

$$
r_{ik} \notin \{n-4, n-3, n-2, n-1\} \qquad (i,k) \in \text{Regional}
$$

Fairness

Each team has x home breaks and y away breaks; x and y should be small

$$\mathrm{gcc}((b_{ij})_{1 \le j \le n-1}, \langle bA, \star, bH \rangle) \qquad 1 \le i \le n$$
$$bA \le maxbA$$
$$bH \le maxbH$$

Fairness

At most 4 consecutive games in SP state, for any SP state team

See below

At most 3 consecutive games in SP state, for any non SP state team

See below

At most 5 consecutive games against G12 teams, for any team

$$\texttt{regular}(\ldots(o_{ij})_{1\leq j\leq 2(n-1)}\ldots,\mathcal{A}) \qquad 1\leq i\leq n$$

Fairness

No consecutive Classic games for any team

$$\texttt{allMinDistance}((r_{ik})_{(i,k)\in \text{Classic}_i}, 2) \qquad 1 \le i \le n$$

Fairness

Perfect matching of paired teams

$$\texttt{regular}(\langle h'_{ik}, h'_{i+1,k}, h'_{i+1,k+1}, h'_{i,k+1}\rangle, \mathcal{A}) \qquad i \in \{1,3,5,\ldots,n-3\}, \; k \in \{i+2, i+4, \ldots, n-1\}$$

More sophisticated generic variable-selection heuristics

Can branch on opponent, home, round, and homeAway variables

- *IBS*: Select variable to maximize impact of that choice on size of search space; select value to minimize that impact
- *maxSD*: Select variable-value to (try to) maximize the proportion of solutions being preserved

Benchmark Instance

The 2010 season of the 1st division of the Brazilian Professional Football League;
a particularly difficult year to schedule

Results so far

- *maxSD* performs much better than the other heuristics
- solved easily (a few seconds) ... without *perfect matching constraints*:

$$\texttt{regular}(\langle h'_{ik}, h'_{i+1,k}, h'_{i+1,k+1}, h'_{i,k+1}\rangle, \mathcal{A})$$

- solved easily (a few seconds) ... without *no regional games at beginning or end constraints*:

$$r_{ik} \notin \{1, 2, 3, n-4, n-3, n-2, n-1\}$$

- with all constraints: nothing after several hours...
- The League uses an IP approach, generating solutions in a few minutes

Objectifs de cette présentation

- Meilleure compréhension de la planification d'horaires de ligues sportives
- C'est un domaine complexe avec une notion de qualité d'horaire difficile à exprimer
- La programmation par contraintes est très appropriée pour résoudre ce problème