Parallel Discrepancy-based Search

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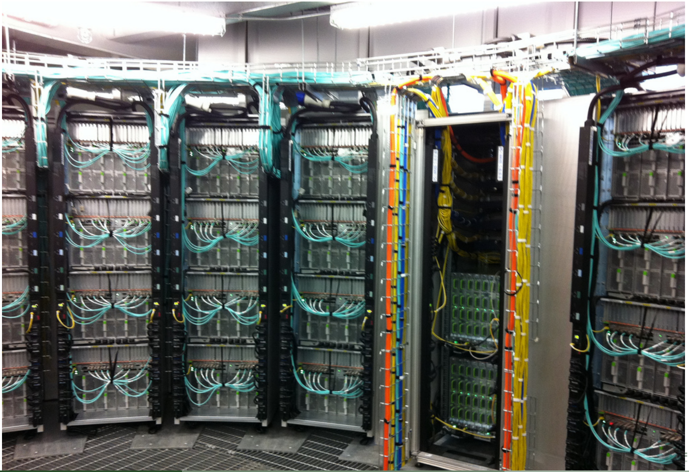
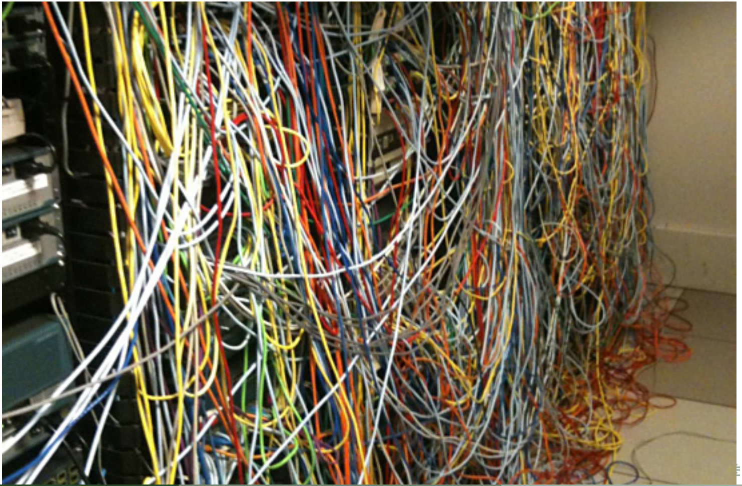
Upsides of Parallelization

- Parallel computing is a growing domain.
- Governments and industries are intensively investing in it.
- It is a huge opportunity for constraint programming.
- It can lead to unprecedented performances.



Downsides of Parallelization

- Parallelization is hard.
- Speedup can stall very fast due to communication.
- It is difficult to reproduce in a parallel environment the search strategies that work well sequentially.

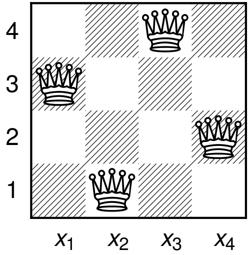


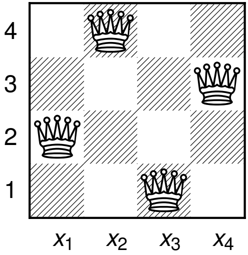
Outline

Constraint programming

- Uses variables with a finite domain;
- Uses specific constraints to modelize the problem:
  - All-different: variables must have different values;
  - Regular: variables must follow a defined regular expression.
- Each constraint includes a specialized algorithm to filter the search space;
- Can include an objective function.

Exemple



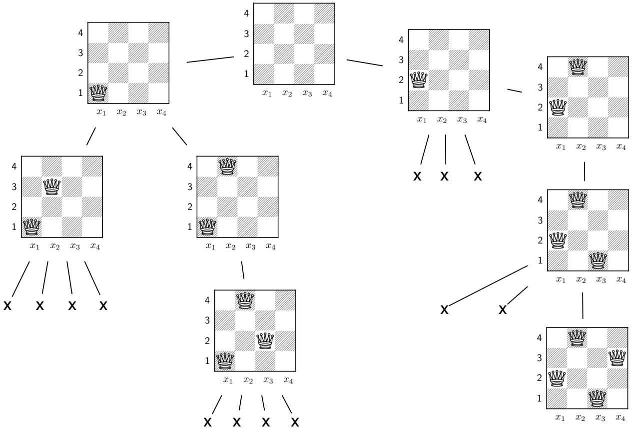


$$\mathrm{dom}(x_1) = \{1, 2, 3, 4\}$$
$$\mathrm{dom}(x_2) = \{1, 2, 3, 4\}$$
$$\mathrm{dom}(x_3) = \{1, 2, 3, 4\}$$
$$\mathrm{dom}(x_4) = \{1, 2, 3, 4\}$$

$$x_i \neq x_j \ \forall i < j$$
$$x_j - x_i \neq j - i \ \forall i < j$$
$$x_j - x_i \neq i - j \ \forall i < j$$

Espace de recherche

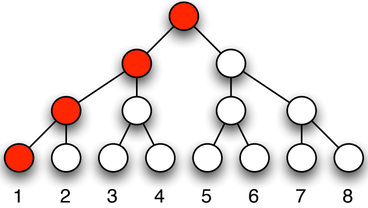
- Ensemble des assignations partielles des variables
  - Cet espace est typiquement représenté par un arbre n-aire.



Outline

Depth-first Search (DFS)

The value ordering heuristic function orders the children of a node from the most likely one to lead to a solution to the least likely one.



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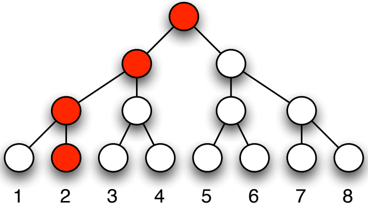

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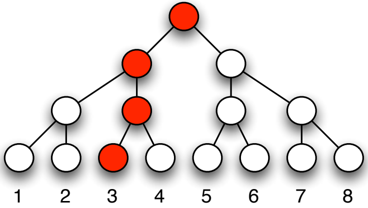

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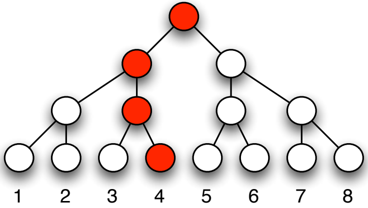

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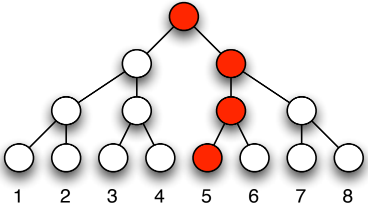

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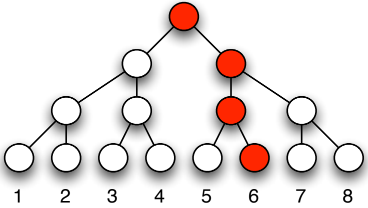

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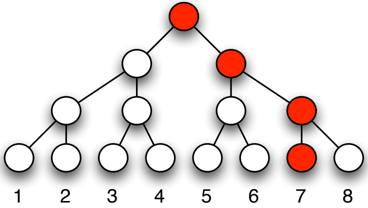

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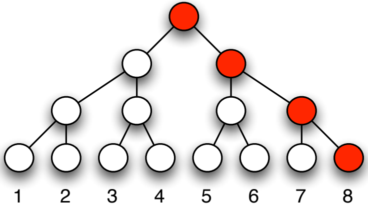
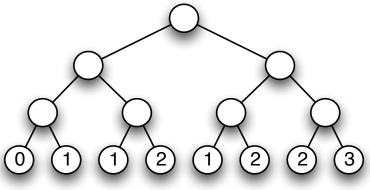


Outline

Limited Discrepancy Search (LDS)

Harvey and Ginsberg (1995)

The value ordering heuristic function orders the children of a node from the most likely one to lead to a solution to the least likely one. A discrepancy is a deviation from the first choice of the value ordering heuristic.



LDS proposes to visit the leaves in increasing order of discrepancy.

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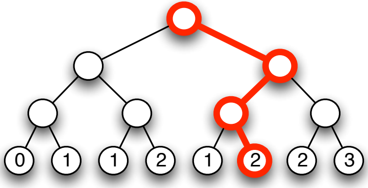
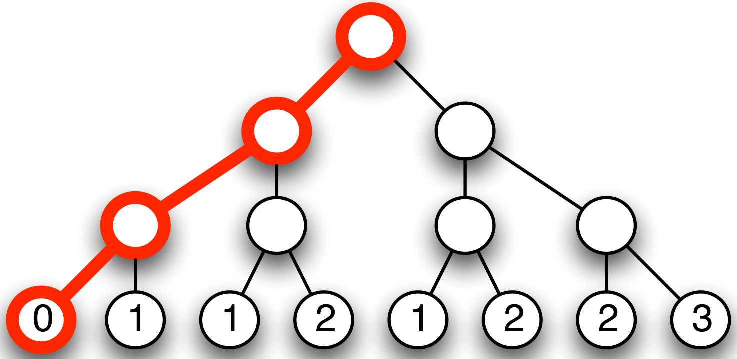
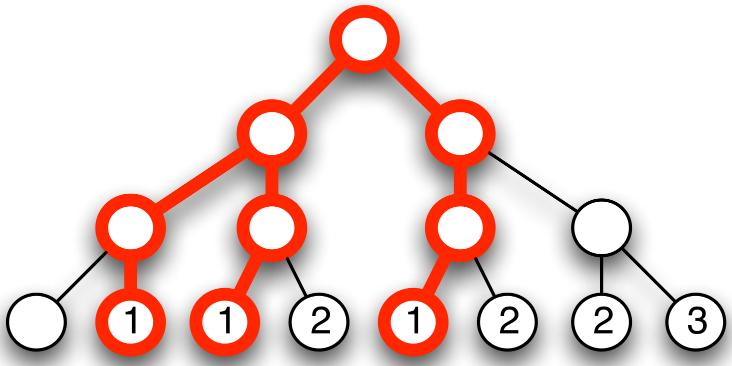


LDS proposes to visit the leaves in increasing order of discrepancy.

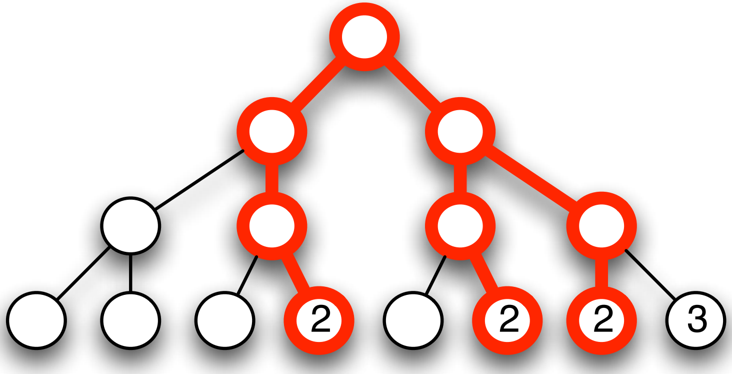
LDS Iterations 0 (0 discrepancy)



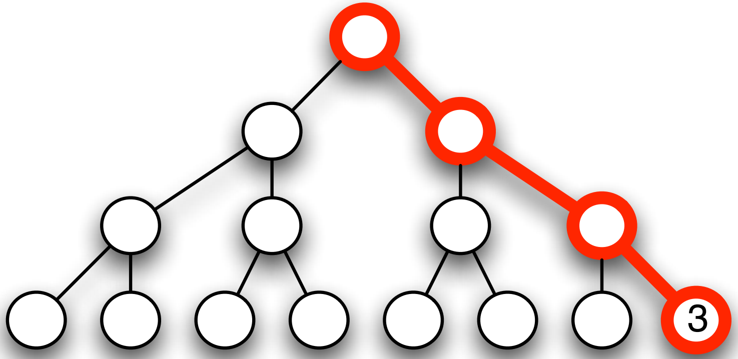
LDS Iterations 1 (1 discrepancy)



LDS Iterations 2 (2 discrepancies)



LDS Iterations 3 (3 discrepancies)



LDS Overhead versus DFS

- Nodes in a binary tree with n variables: $2 \cdot 2^n - 1$
- Node visits with a DFS: $2 \cdot 2^n - 1$
- Node visits with a LDS: $4 \cdot 2^n - n - 3$
- By the time DFS visits the entire tree, LDS will visit half of the leaves.
- The leaves that LDS visits are those that have fewer than n/2 discrepancies.
- If the value ordering heuristic is as good as a random heuristic, LDS finds a solution by the time DFS completes.

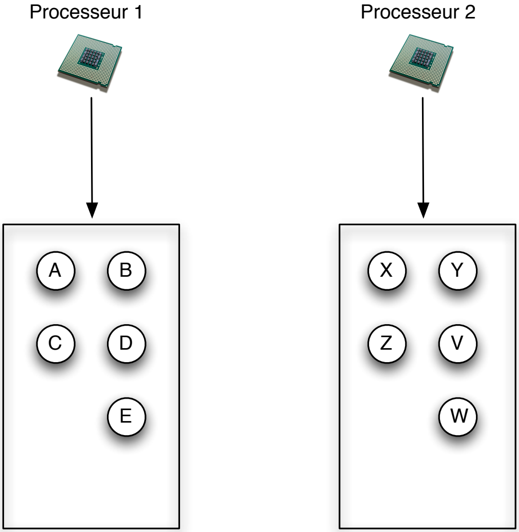
Outline

Portfolio

- Il s'agît d'une course entre les processeurs pour obtenir une solution.
- Chaque processeur recherche une solution avec un algorithme qui lui est propre
- Les processeurs peuvent s'aider en échangeant des informations.
- Il est difficile d'étendre à un nombre arbitraire de processeurs.
  - Il faut définir des paramètres de recherche pour chaque processeur.
  - Ces paramètres sont souvent établis manuellement.
  - La paramétrisation est fortement lié au problème à résoudre.

Work-stealing

Xie et Davenport



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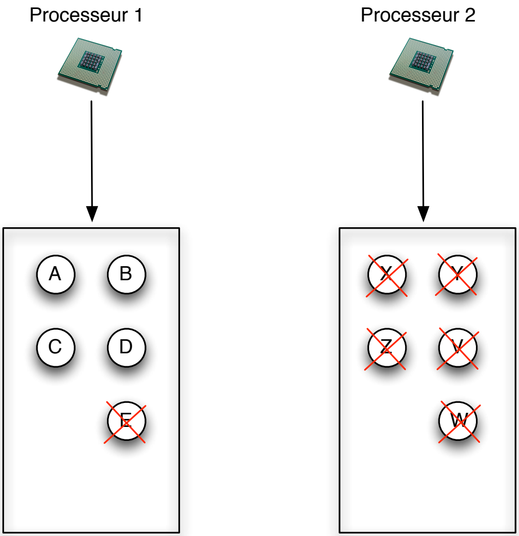

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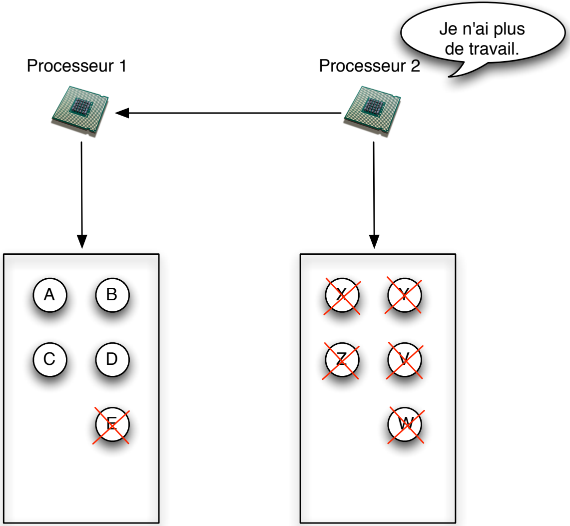

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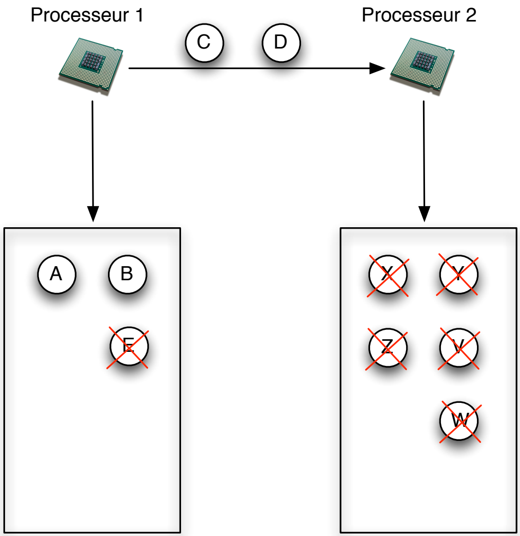

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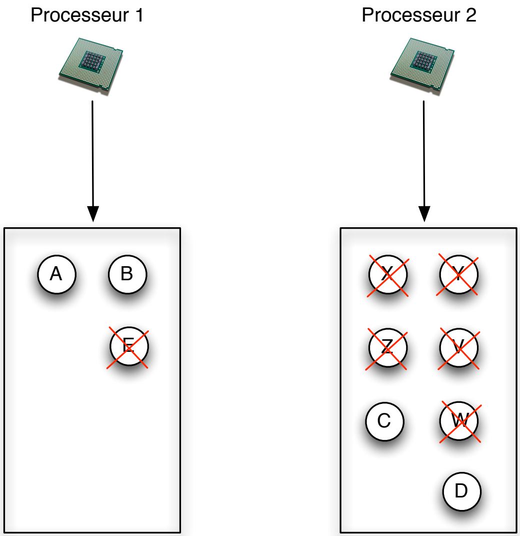

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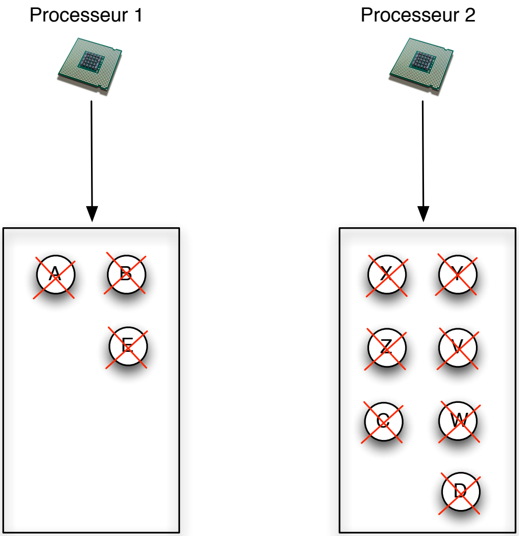


Division de l'espace de recherche

- Approche très courante
- Divise théoriquement la charge de travail également
- Peu probable que les sous-arbres soient de même taille
  - Ne peut pas être utilisé sur un grand nombre de processeurs
  - Problème NP-Difficile
- Si les sous-arbres ne sont pas de même taille
  - Les processeurs n'auront pas la même charge de travail
  - La durée totale de la tâche sera plus longue

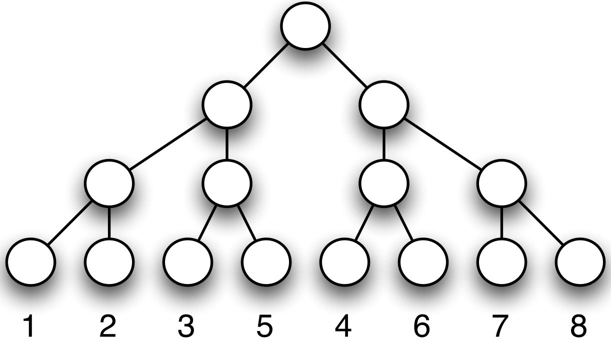
Outline

Our Objectives

- Having a search strategy that performs well sequentially, we want to parallelize it with four main goals in mind:
  - Search strategy preservation
  - Workload balancing
  - Robustness
  - No communication

Intuition behind PDS

- Each leaf is assigned to a processor.
- The leaves are implicitly assigned in a round-robin fashion.
- To split the task, let's say that p_1 takes odd leaves and p_2 the even leaves.



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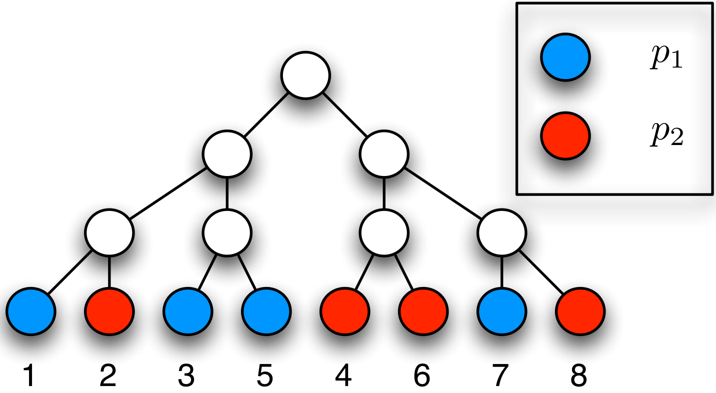
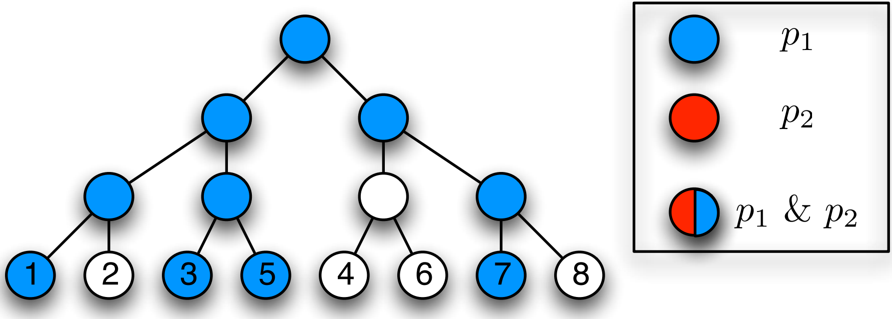
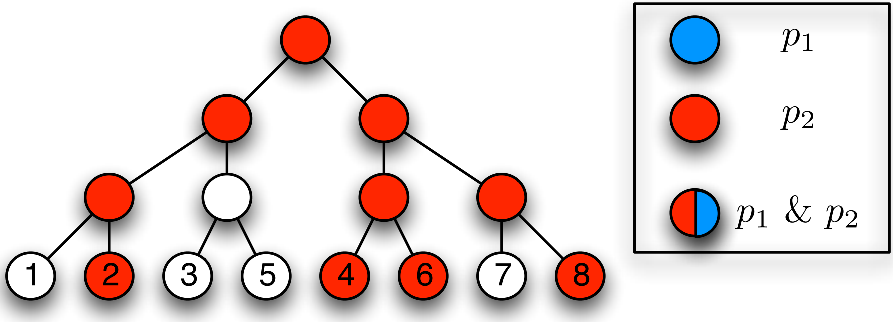


PDS overview



PDS overview



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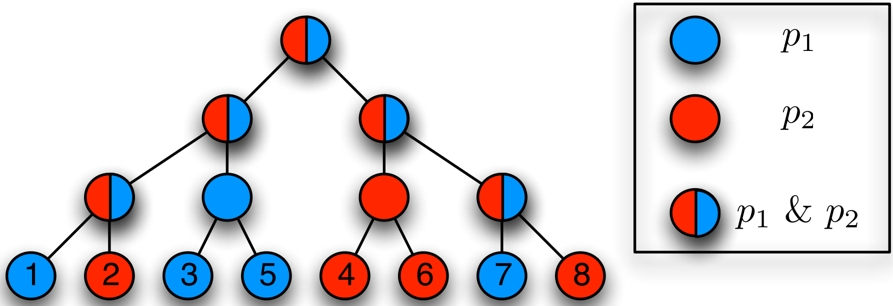
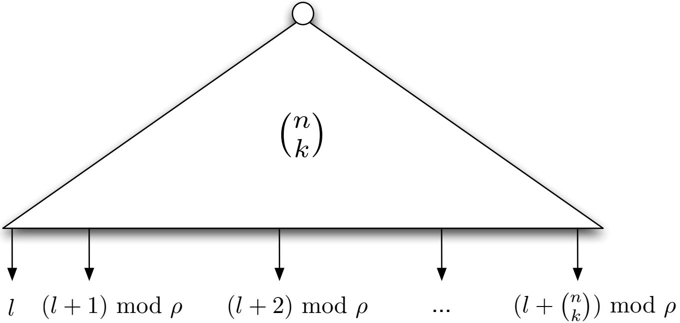
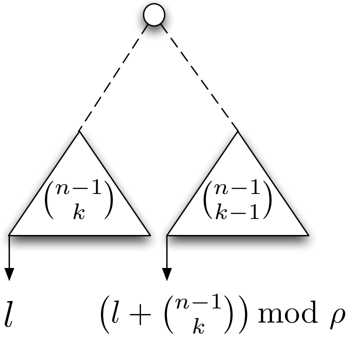


Implementing PDS

- p : its processor id
- ρ : the number of processors
- n : the number of variables in the subtree
- k: the number of required discrepancies
- l : the processor id implicitly assigned to the leftmost leaf
- The processor will branch in the subtree if $(p - l) \bmod \rho < \binom{n}{k}$



Branching in PDS

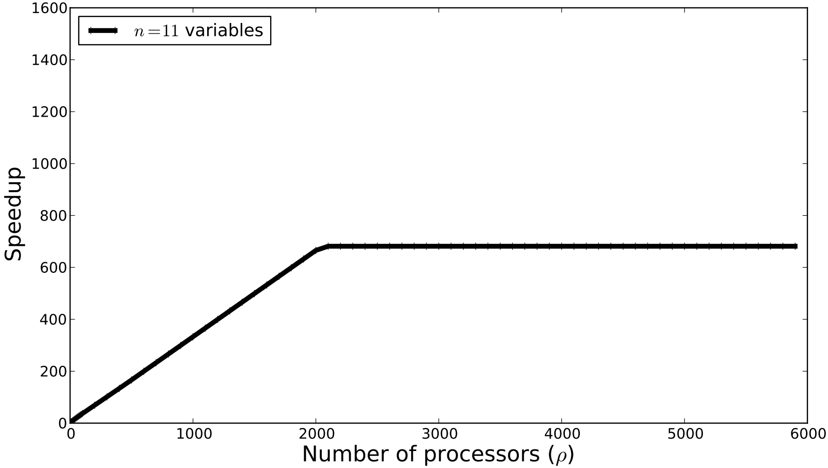


Outline

PDS overhead versus LDS

Processors	Node visits	Overhead	Speedup
1	$4 \cdot 2^n - n - 3$	-	-
2	$5 \cdot 2^n - 2n - 4$	25%	1.61
3	$5.75 \cdot 2^n - 3n - 5$	43%	2.08
...	...	...	...
ρ	$2^n + 2^n \sum_{i=1}^{n} \sum_{k=0}^{i} \frac{1}{2^i} \min(\rho, \binom{i}{k})$		

Speedup for some number of processors



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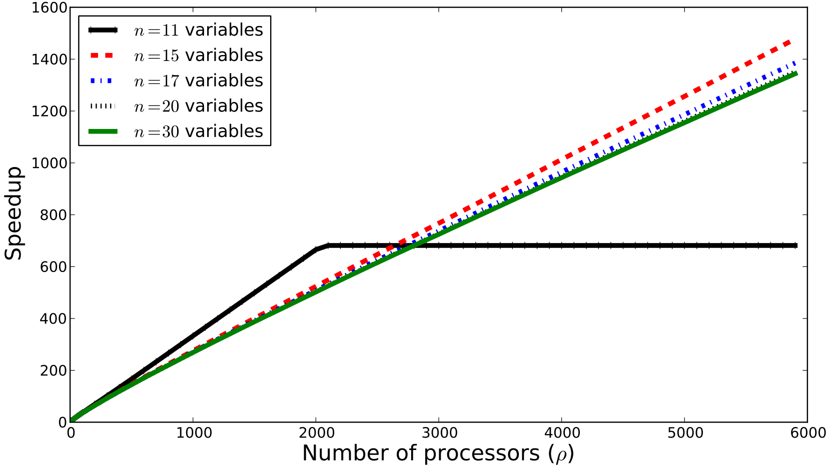
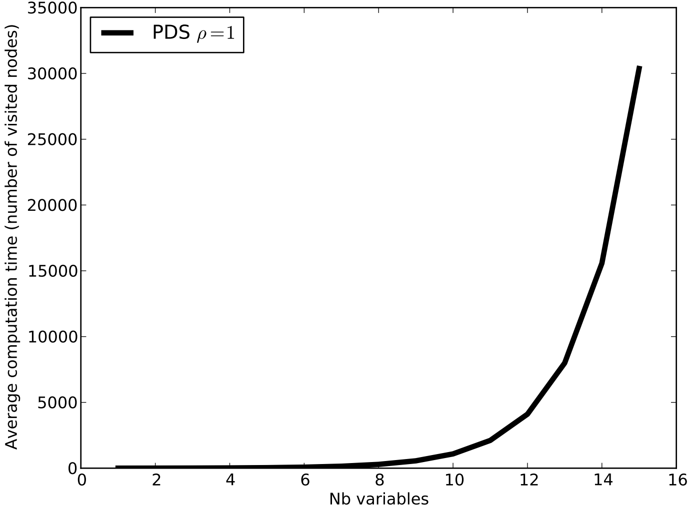


Average computation time to find a solution according to the number of variables



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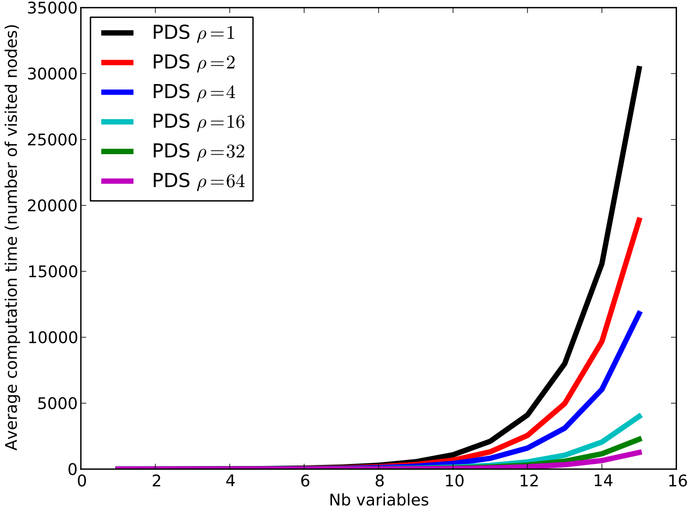
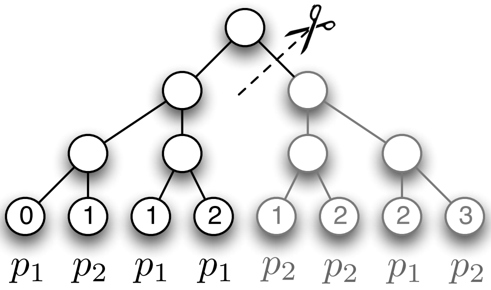


Balancing

Theorem

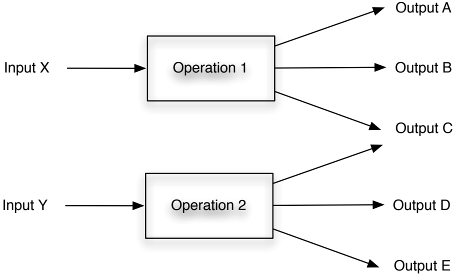
Let n be the number of variables in the problem. If a branch is cut from the search tree, the number of leaves removed from the workload of each processor differs by at most n.



Outline

Planning and scheduling wood finishing operations

- We minimize order lateness.
- The production is done with one-to-many operations.
- There is multiple ways to produce the same product.



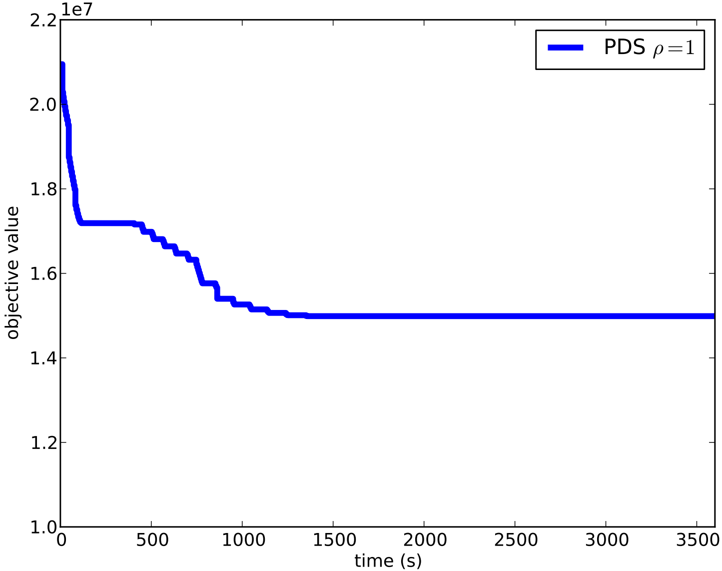
- There are setup constraints that restrict sequencing of the operations.
- This is a problem for which LDS was efficient thanks to a specialized value ordering heuristic (Gaudreault et.al (2010)).

Instances from a Canadian Forest-Product Company

We worked with industrial instances:

- 65,142 variables
- 50,238 constraints
- 42 discrete decision variables whose domains have cardinality 6
  - Used LDS/PDS to fix those variables.
  - Once the discrete decision variables are known, the remaining continuous variables define a linear program that can be easily solved to optimality.
- 4200 continuous decision variables
  - Linear program solved with Cplex.

Results



1e7
2.2
2.0
1.8
1.6
1.4
1.2
1.0
objective value
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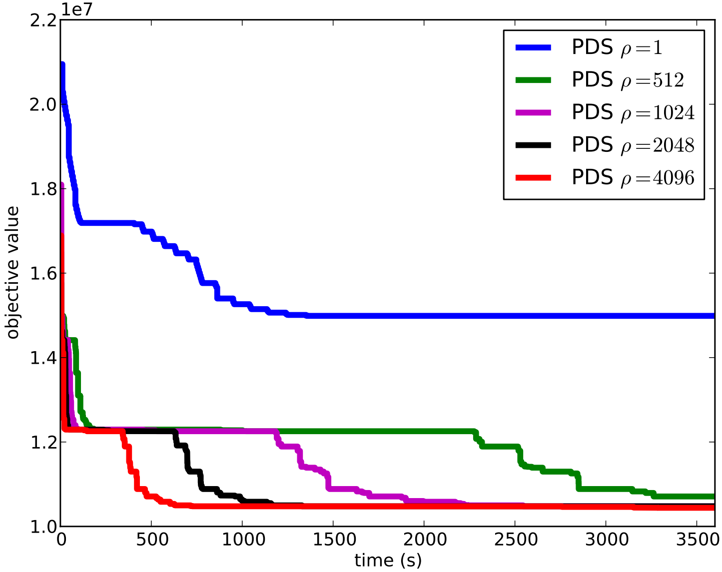
500
1000
1500
2000
2500
3000
3500
time (s)
PDS $\rho = 1$

Results



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time (s)
PDS $\rho = 1$
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PDS $\rho = 2048$
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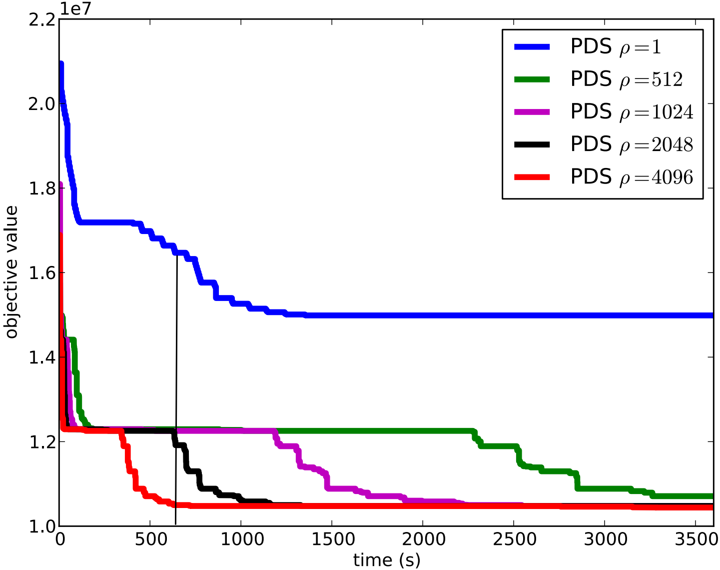

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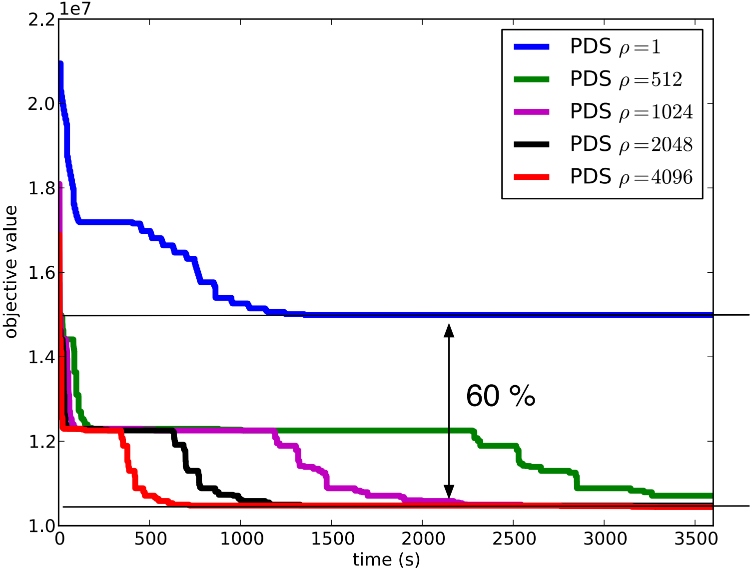

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60 %

Conclusion

- Theoretical analysis of DFS versus LDS
- Parallel Discrepancy-based Search (PDS)
  - LDS search strategy preservation
  - No communication
  - Intrinsic load balancing
  - Robust to hardware failures
- Theoretical analysis of this parallelization.
- Experimental results on large industrial instances with up to 4096 processors.