| Reflected in $\theta = 0$[3] | Reflected in $\theta = \pi/2$ (co-function identities)[4] | Reflected in $\theta = \pi$ |
|---|---|---|
| $\sin(-\theta) = -\sin\theta$ | $\sin(\frac{\pi}{2} - \theta) = +\cos\theta$ | $\sin(\pi - \theta) = +\sin\theta$ |
| $\cos(-\theta) = +\cos\theta$ | $\cos(\frac{\pi}{2} - \theta) = +\sin\theta$ | $\cos(\pi - \theta) = -\cos\theta$ |
| $\tan(-\theta) = -\tan\theta$ | $\tan(\frac{\pi}{2} - \theta) = +\cot\theta$ | $\tan(\pi - \theta) = -\tan\theta$ |
| $\csc(-\theta) = -\csc\theta$ | $\csc(\frac{\pi}{2} - \theta) = +\sec\theta$ | $\csc(\pi - \theta) = +\csc\theta$ |
| $\sec(-\theta) = +\sec\theta$ | $\sec(\frac{\pi}{2} - \theta) = +\csc\theta$ | $\sec(\pi - \theta) = -\sec\theta$ |
| $\cot(-\theta) = -\cot\theta$ | $\cot(\frac{\pi}{2} - \theta) = +\tan\theta$ | $\cot(\pi - \theta) = -\cot\theta$ |

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

| Double-angle formulae[ | | | |
|---|---|---|---|
| $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ | $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ | $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ | $\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$ |

| Sine | Cosine | Other |
|---|---|---|
| $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ | $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ | $\sin^2\theta\cos^2\theta = \frac{1-\cos 4\theta}{8}$ |
| $\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$ | $\cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$ | $\sin^3\theta\cos^3\theta = \frac{3\sin 2\theta - \sin 6\theta}{32}$ |

| Shift by π/2 | Shift by π<br>Period for tan and cot[5] | Shift by 2π<br>Period for sin, cos, csc and sec[6] |
|---|---|---|
| $\sin(\theta + \frac{\pi}{2}) = +\cos\theta$ | $\sin(\theta + \pi) = -\sin\theta$ | $\sin(\theta + 2\pi) = +\sin\theta$ |
| $\cos(\theta + \frac{\pi}{2}) = -\sin\theta$ | $\cos(\theta + \pi) = -\cos\theta$ | $\cos(\theta + 2\pi) = +\cos\theta$ |
| $\tan(\theta + \frac{\pi}{2}) = -\cot\theta$ | $\tan(\theta + \pi) = +\tan\theta$ | $\tan(\theta + 2\pi) = +\tan\theta$ |
| $\csc(\theta + \frac{\pi}{2}) = +\sec\theta$ | $\csc(\theta + \pi) = -\csc\theta$ | $\csc(\theta + 2\pi) = +\csc\theta$ |
| $\sec(\theta + \frac{\pi}{2}) = -\csc\theta$ | $\sec(\theta + \pi) = -\sec\theta$ | $\sec(\theta + 2\pi) = +\sec\theta$ |
| $\cot(\theta + \frac{\pi}{2}) = -\tan\theta$ | $\cot(\theta + \pi) = +\cot\theta$ | $\cot(\theta + 2\pi) = +\cot\theta$ |

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

| | |
|---|---|
| $\sin[\arccos(x)] = \sqrt{1-x^2}$ | $\tan[\arcsin(x)] = \frac{x}{\sqrt{1-x^2}}$ |
| $\sin[\arctan(x)] = \frac{x}{\sqrt{1+x^2}}$ | $\tan[\arccos(x)] = \frac{\sqrt{1-x^2}}{x}$ |
| $\cos[\arctan(x)] = \frac{1}{\sqrt{1+x^2}}$ | $\cot[\arcsin(x)] = \frac{\sqrt{1-x^2}}{x}$ |
| $\cos[\arcsin(x)] = \sqrt{1-x^2}$ | $\cot[\arccos(x)] = \frac{x}{\sqrt{1-x^2}}$ |

| | |
|---|---|
| Sine | $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$[7][8] |
| Cosine | $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$[8][9] |
| Tangent | $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$[8][10] |

$$\sin\theta \approx \theta, \cos\theta \approx 1 \quad \text{si} \quad \theta \ll 1 \text{ rad}$$

half-angles formulae

$$\sin\frac{\theta}{2} = \operatorname{sgn}\left(2\pi - \theta + 4\pi\left\lfloor\frac{\theta}{4\pi}\right\rfloor\right)\sqrt{\frac{1-\cos\theta}{2}}$$

$$\left(\text{or } \sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}\right)$$

$$\cos\frac{\theta}{2} = \operatorname{sgn}\left(\pi + \theta + 4\pi\left\lfloor\frac{\pi - \theta}{4\pi}\right\rfloor\right)\sqrt{\frac{1+\cos\theta}{2}}$$

$$\left(\text{or } \cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}\right)$$

$$\tan\frac{\theta}{2} = \csc\theta - \cot\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

$$\tan\frac{\eta+\theta}{2} = \frac{\sin\eta + \sin\theta}{\cos\eta + \cos\theta}$$

$$\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \sec\theta + \tan\theta$$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\tan(\theta/2)}{1+\tan(\theta/2)}$$

$$\tan\tfrac{1}{2}\theta = \frac{\tan\theta}{1+\sqrt{1+\tan^2\theta}} \quad \text{for} \quad \theta \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right)$$

$$\cot\frac{\theta}{2} = \csc\theta + \cot\theta = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

| Product-to-sum[23] |
|---|
| $\cos\theta\cos\varphi = \frac{\cos(\theta-\varphi) + \cos(\theta+\varphi)}{2}$ |
| $\sin\theta\sin\varphi = \frac{\cos(\theta-\varphi) - \cos(\theta+\varphi)}{2}$ |
| $\sin\theta\cos\varphi = \frac{\sin(\theta+\varphi) + \sin(\theta-\varphi)}{2}$ |
| $\cos\theta\sin\varphi = \frac{\sin(\theta+\varphi) - \sin(\theta-\varphi)}{2}$ |
| $\tan\theta\tan\varphi = \frac{\cos(\theta-\varphi) - \cos(\theta+\varphi)}{\cos(\theta-\varphi) + \cos(\theta+\varphi)}$ |

| Sum-to-product[24] |
|---|
| $\sin\theta \pm \sin\varphi = 2\sin\left(\frac{\theta \pm \varphi}{2}\right)\cos\left(\frac{\theta \mp \varphi}{2}\right)$ |
| $\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta + \varphi}{2}\right)\cos\left(\frac{\theta - \varphi}{2}\right)$ |
| $\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta + \varphi}{2}\right)\sin\left(\frac{\theta - \varphi}{2}\right)$ |

## cinématique directe robot 2D

$$x(t) = \int_0^t V(t)\cos(\theta(t))dt$$

$$y(t) = \int_0^t V(t)\sin(\theta(t))dt$$

$$\theta(t) = \int_0^t \omega(t)dt$$

## Conduite différentielle

$$R = \frac{l}{2}\frac{v_l + v_r}{v_r - v_l}, \quad \omega = \frac{v_r - v_l}{l}, \quad V = \frac{v_r + v_l}{2}$$

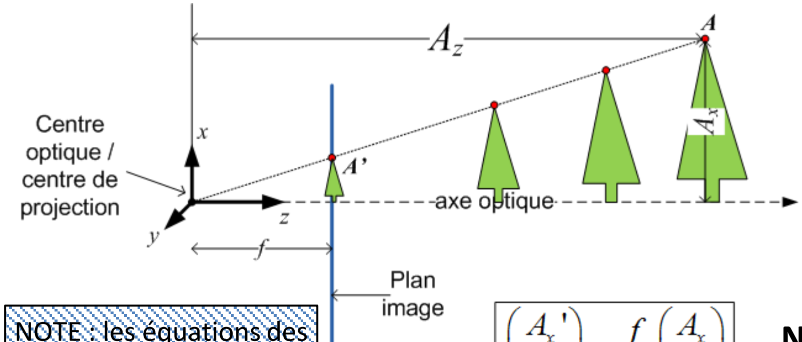
$l$: distance entre roues

$v_l, v_r$ : vitesse linéare des roues

R : position ICC p/r milieu entre roues

V : vitesse linéaire du robot

ω: vitesse angulaire du robot




NOTE : les équations des filtres de Kalman et Kalman étendu sont données dans l'examen.

$$\begin{pmatrix} A_x' \\ A_y' \end{pmatrix} = \frac{f}{A_z}\begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

**Distribution normale**

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

**Règle de Bayes**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

**Fusion capteur**

$$z_3 = (1-w)z_1 + wz_2$$

$$w = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

**Règles des dérivées**

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$(x^n)' = nx^{n-1}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(\ln|x|)' = \frac{1}{x}$$

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

**Note : les parties hachurées ne concernent pas l'examen de mi session**

**disparité stéréo**

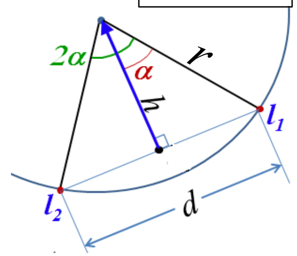
$$d = A'_{X2} - A'_{X1} = \frac{f\,b}{A_z}$$

**Caméra perspective**

$$\underset{A'}{\begin{bmatrix} Ax' \\ Ay' \\ 1 \end{bmatrix}} \sim \underset{P}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix}} \underset{A}{\begin{bmatrix} Ax \\ Ay \\ Az \\ 1 \end{bmatrix}}$$

**angle inscrit et angle au centre**

$$\frac{h}{d} = \frac{1}{2\tan\alpha} \qquad r = \frac{d}{2\sin\alpha}$$




**Each trigonometric function in terms of the other five.**

| in terms of | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\csc\theta$ | $\sec\theta$ | $\cot\theta$ |
|---|---|---|---|---|---|---|
| $\sin\theta =$ | $\sin\theta$ | $\pm\sqrt{1-\cos^2\theta}$ | $\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ | $\frac{1}{\csc\theta}$ | $\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$ | $\pm\frac{1}{\sqrt{1+\cot^2\theta}}$ |
| $\cos\theta =$ | $\pm\sqrt{1-\sin^2\theta}$ | $\cos\theta$ | $\pm\frac{1}{\sqrt{1+\tan^2\theta}}$ | $\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$ | $\frac{1}{\sec\theta}$ | $\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$ |
| $\tan\theta =$ | $\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ | $\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ | $\tan\theta$ | $\pm\frac{1}{\sqrt{\csc^2\theta-1}}$ | $\pm\sqrt{\sec^2\theta-1}$ | $\frac{1}{\cot\theta}$ |
| $\csc\theta =$ | $\frac{1}{\sin\theta}$ | $\pm\frac{1}{\sqrt{1-\cos^2\theta}}$ | $\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$ | $\csc\theta$ | $\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$ | $\pm\sqrt{1+\cot^2\theta}$ |
| $\sec\theta =$ | $\pm\frac{1}{\sqrt{1-\sin^2\theta}}$ | $\frac{1}{\cos\theta}$ | $\pm\sqrt{1+\tan^2\theta}$ | $\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$ | $\sec\theta$ | $\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ |
| $\cot\theta =$ | $\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ | $\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$ | $\frac{1}{\tan\theta}$ | $\pm\sqrt{\csc^2\theta-1}$ | $\pm\frac{1}{\sqrt{\sec^2\theta-1}}$ | $\cot\theta$ |

**Jacobienne de** $\begin{pmatrix} f_1(x_1,\ldots,x_n) \\ \vdots \\ f_m(x_1,\ldots,x_n) \end{pmatrix}$ **est** $J_F(P) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$

$$T = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

cote(p) = p/(1-p)    p = cote/(1+cote)    $\text{cote}(c,t) = \frac{p(z|c)}{p(z|\bar{c})}\text{cote}(c,t-1)$

**Statistiques :**

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\,\text{Cov}(X,Y).$$