Beyond Finite Domains: the All-Different and Global Cardinality Constraints

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ABSTRACT

We describe how the propagators for the ALL-DIFFERENT constraint and the global cardinality constraint (GCC) can be generalized to prune variables whose domains are not just simple finite domains. We show, for example, how they can propagate set variables, multiset variables and variables which represent tuples of values. Experiments show that such propagators can be beneficial in practice, especially when the domains are large.

1 Introduction

- Most CSPs are encoded with integer variables.
- There exists a multitude of variable types.
  - Set variables $X = \{1, 4, 6\}$
  - Multi-set variables $X = \{1, 1, 4, 4, 4, 6\}$
  - Ordered tuple variables $X = [1, 4, 2, 2]$
  - String variables $X =$ "Hello World!"
  - …
- Structured variables
  - reduce the memory space;
  - improve efficiency of constraint propagators;
  - and ease data abstraction.

2 Generalizing Propagators

Propagators for integer variables can be used on more complex variables.

2.1 Example

We propagate the ALL-DIFFERENT constraint on the following set variables.

$$\{\} \subseteq S_1, S_2, S_3, S_4 \subseteq \{1, 2\}$$
$$\{\} \subseteq S_5, S_6 \subseteq \{2, 3\}$$

1) We expand variable domains.

$$S_1, S_2, S_3, S_4 \in \{\{\}, \{1\}, \{2\}, \{1, 2\}\}$$
$$S_5, S_6 \in \{\{\}, \{2\}, \{3\}, \{2, 3\}\}$$

2) We enforce GAC on ALL-DIFFERENT.

$$S_1, S_2, S_3, S_4 \in \{\{\}, \{1\}, \{2\}, \{1, 2\}\}$$
$$S_5, S_6 \in \{\{3\}, \{2, 3\}\}$$

3) We convert the domains to their original form.

$$\{\} \subseteq S_1, S_2, S_3, S_4 \subseteq \{1, 2\}$$
$$\{3\} \subseteq S_5, S_6 \subseteq \{2, 3\}$$

2.2 Tractability

- The speed of some propagators depends on the size of the domains.
  - E.g.: ALL-DIFFERENT runs in $O(n^{1.5}d)$.
- Complex variable structures often have large domain sizes.
  - $\text{dom}(A) = \{S \mid lb \subseteq S \subseteq ub\}$
  - $|dom(A)| = 2^{|ub - lb|}$
- Variables with large domains must be handled with care.

3 Adapting the All-Different Propagator

3.1 Principle

- Suppose S is a variable s.t. $|\text{dom}(S)| > n$. Then S does not constrain other variables since it can be assigned to a value different than the other variables.
- Run the ALL-DIFFERENT propagator over "small" variables.
- Remove values that must be assigned to small domain variables from the larger domains. Resulting complexity: $O(n^{2.5})$.

3.2 Requirements from our Adaptation

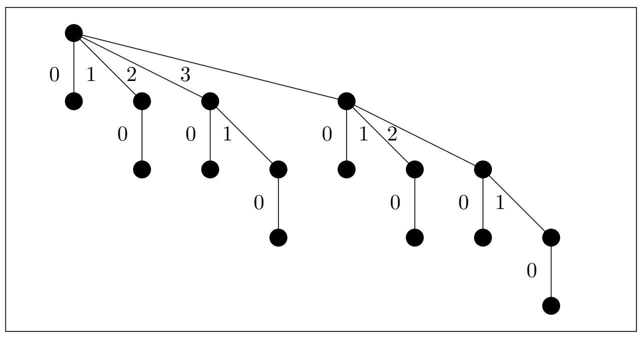
The variable domains must be:

1. Efficiently countable;
2. Efficiently enumerable;
3. Efficiently computed from an enumeration of values.

4 Sets

4.1 Domains with Bounds

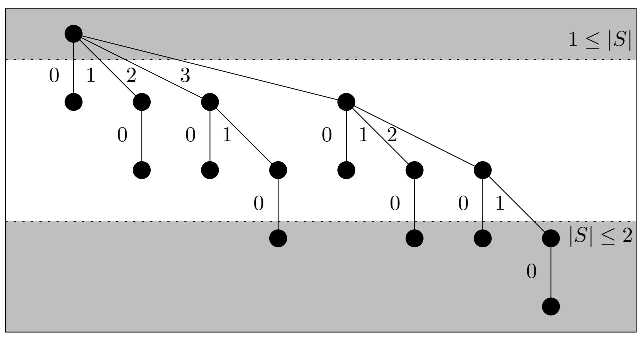
- $\text{dom}(X) = \{S \mid lb \subseteq S \subseteq ub\}$
- $|\text{dom}(X)| = 2^{|ub| - |lb|}$
- Enumeration: Depth first search in a binomial tree.



$\{\} \subseteq S \subseteq \{0, 1, 2, 3\}$

4.2 Set Variables with Constrained Size

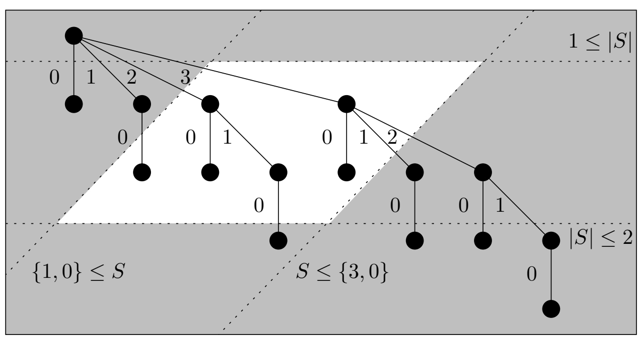
- $\text{dom}(S_i) = \{S \mid lb \subseteq S \subseteq ub, a \leq |S| \leq b\}$
- $|\text{dom}(S_i)| = \sum_{j=a}^{b} \binom{|ub - lb|}{j - |lb|}$



$\{\} \subseteq S \subseteq \{0, 1, 2, 3\}, 1 \leq |S| \leq 2$

4.3 Set Variables with Lexicographical Constraints

- $\text{dom}(S_i) = \{S \mid lb \subseteq S \subseteq ub, a \leq |S| \leq b, l \leq S \leq u\}$
- $|dom(S_i)|$ can be computed using a binary representation of sets l and u.



$\{\} \subseteq S \subseteq \{0, 1, 2, 3\}, 1 \leq |S| \leq 2, \{1, 0\} \leq S \leq \{3, 0\}$

5 Tuples

5.1 Factored Encoding

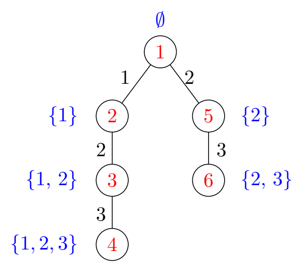
- One variable per component: $[t_1, t_2, \ldots, t_n]$.
- To impose an ALL-DIFFERENT on all tuples, we map tuples to integer variables.
  - Map a pair to an integer: $t = t_1 n + t_2$.
  - Post ALL-DIFFERENT on integer variables.
  - Issue: channeling is either slow or inefficient.

5.2 Component Encoding

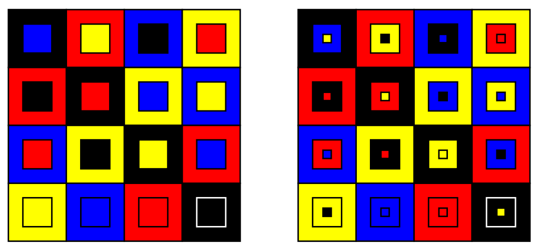
- Component Encoding
  - $\text{dom}(t) = \text{dom}(t_1) \times \ldots \times \text{dom}(t_n)$.
  - $|\text{dom}(t)| = \prod_{i=1}^{n} |\text{dom}(t_i)|$.
  - Enumeration: done in $O(|\text{dom}(t)|)$ steps.
- Lexicographical bounds can be added as for sets.

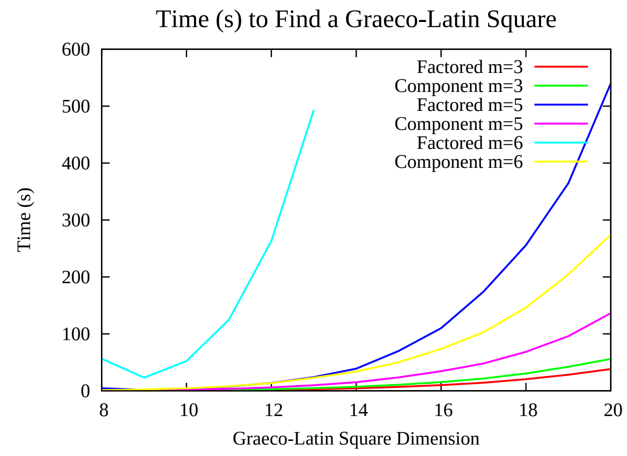
6 Indexing Domain Values

- Propagators often store information about a value v in a table entry $T[v]$.
- Non integer values need to be mapped to an integer.



7 Experiments: Graeco-Latin Squares





- The component representation offers better performances for large tuples.

8 Conclusion

- We have shown that propagators for integer variables can be adapted for more complex variables.
- Variable domains must be countable, enumerable, and computable from an enumeration of values.
- Results also apply for the Global Cardinality Constraint.
- We also studied multi-sets.