

## Réponse à la série d'exercices #7

### Optimisations et analyses statiques

$$1. \text{ (a)} \quad \begin{array}{cccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & \delta_{13} & \checkmark \\ \delta_{17} & \checkmark & \delta_{18} & \checkmark & \delta_{19} & \checkmark & \delta_{20} & \checkmark & \delta_{21} & \checkmark \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{\#f, P_8\} & \alpha_2 = \{\lambda_5\} & \alpha_3 = \{\lambda_4\} & \alpha_4 = \{\lambda_4\} \\ \alpha_5 = \{\lambda_5\} & \alpha_6 = \{\#f, P_8\} & \alpha_7 = \{\#f, P_{18}\} & \alpha_8 = \{P_8\} \\ \alpha_9 = \{\#f\} & \alpha_{10} = \{\#f, P_{18}\} & \alpha_{11} = \{\#f, P_8\} & \alpha_{12} = \{\lambda_5\} \\ \alpha_{13} = \{\lambda_4\} & \alpha_{14} = \{\#f\} & \alpha_{15} = \{\#f, P_{18}\} & \alpha_{16} = \{\#f\} \\ \alpha_{17} = \{\#f\} & \alpha_{18} = \{P_{18}\} & \alpha_{19} = \{\#f\} & \alpha_{20} = \{\#f\} \\ \alpha_{21} = \{\#f\} & & & \\ \alpha_A = \{\lambda_4\} & \alpha_m = \{\#f, P_{18}\} & \alpha_n = \{\#f\} & \end{array}$$

$$(b) \quad \begin{array}{cccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & \delta_{13} & \checkmark \\ & & & & & & & & \delta_{14} & \checkmark \\ & & & & & & & & \delta_{15} & \checkmark \\ & & & & & & & & \delta_{16} & \checkmark \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{P_{11}\} & \alpha_2 = \{\lambda_4\} & \alpha_3 = \{\lambda_3\} & \alpha_4 = \{\lambda_4\} \\ \alpha_5 = \{P_{11}\} & \alpha_6 = \{\lambda_{10}\} & \alpha_7 = \{P_{11}\} & \alpha_8 = \{\lambda_{10}\} \\ \alpha_9 = \{P_{14}\} & \alpha_{10} = \{\lambda_{10}\} & \alpha_{11} = \{P_{11}\} & \alpha_{12} = \{\#f\} \\ \alpha_{13} = \{P_{11}, P_{14}\} & \alpha_{14} = \{P_{14}\} & \alpha_{15} = \{\#f\} & \alpha_{16} = \{\#f\} \\ \alpha_f = \{\lambda_{10}\} & \alpha_x = \{P_{14}\} & \alpha_y = \{P_{11}, P_{14}\} & \end{array}$$

$$(c) \quad \begin{array}{cccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & \delta_{13} & \checkmark \\ & & & & & & & & \delta_{14} & \checkmark \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{\#f\} & \alpha_2 = \{\lambda_7\} & \alpha_3 = \{\lambda_3\} & \alpha_4 = \{\lambda_7\} \\ \alpha_5 = \{\#f\} & \alpha_6 = \emptyset & \alpha_7 = \{\lambda_7\} & \alpha_8 = \{\#f\} \\ \alpha_9 = \{\#f\} & \alpha_{10} = \{P_{11}\} & \alpha_{11} = \{P_{11}\} & \alpha_{12} = \{\#f\} \\ \alpha_{13} = \{\#f\} & \alpha_{14} = \{\#f\} & & \\ \alpha_z = \{\#f\} & \alpha_x = \{\#f\} & & \end{array}$$

$$(d) \quad \begin{array}{cccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & \delta_{13} & \checkmark \\ & & & & & & & & \delta_{14} & \checkmark \\ & & & & & & & & \delta_{15} & \checkmark \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{\#f\} & \alpha_2 = \{\lambda_2\} & \alpha_3 = \{\#f\} & \alpha_4 = \{\#f\} \\ \alpha_5 = \emptyset & \alpha_6 = \emptyset & \alpha_7 = \emptyset & \alpha_8 = \{\#f\} \\ \alpha_9 = \{\lambda_{13}\} & \alpha_{10} = \{P_{10}\} & \alpha_{11} = \{\#f\} & \alpha_{12} = \{\#f\} \\ \alpha_{13} = \{\lambda_{13}\} & \alpha_{14} = \{\#f\} & \alpha_{15} = \{P_{10}\} & \\ \alpha_f = \{\lambda_{13}\} & \alpha_x = \{P_{10}\} & & \end{array}$$

$$(e) \quad \begin{array}{ccccccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark & \delta_6 & \checkmark & \delta_7 & \checkmark & \delta_8 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & \delta_{13} & \checkmark & \delta_{14} & \checkmark & \delta_{15} & \checkmark & \delta_{16} & \checkmark \\ \delta_{17} & \checkmark & \delta_{18} & \checkmark \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{\#f\} & \alpha_2 = \{\lambda_2\} & \alpha_3 = \{\#f\} & \alpha_4 = \{\lambda_4\} \\ \alpha_5 = \{\#f\} & \alpha_6 = \{\lambda_8, P_{14}\} & \alpha_7 = \{\lambda_{17}\} & \alpha_8 = \{\lambda_8\} \\ \alpha_9 = \{\#f\} & \alpha_{10} = \{\#f\} & \alpha_{11} = \{\#f\} & \alpha_{12} = \{\lambda_8, P_{14}\} \\ \alpha_{13} = \{\lambda_{17}\} & \alpha_{14} = \{P_{14}\} & \alpha_{15} = \{\#f\} & \alpha_{16} = \{\#f\} \\ \alpha_{17} = \{\lambda_{17}\} & \alpha_{18} = \{\lambda_8, P_{14}\} & & \\ \alpha_f = \{\lambda_{17}\} & \alpha_z = \{\#f\} & \alpha_y = \{\#f\} & \alpha_x = \{\lambda_8, P_{14}\} \end{array}$$

$$(f) \quad \begin{array}{ccccccccccccc} \delta_1 & \checkmark & \delta_2 & \checkmark & \delta_3 & \checkmark & \delta_4 & \checkmark & \delta_5 & \checkmark & \delta_6 & \checkmark & \delta_7 & \checkmark & \delta_8 & \checkmark \\ \delta_9 & \checkmark & \delta_{10} & \checkmark & \delta_{11} & \checkmark & \delta_{12} & \checkmark & & & & & & & \end{array}$$

$$\begin{array}{llll} \alpha_1 = \{\#f, \lambda_8\} & \alpha_2 = \{\lambda_2\} & \alpha_3 = \{\#f, \lambda_8\} & \alpha_4 = \{\#f, \lambda_8\} \\ \alpha_5 = \{\lambda_8\} & \alpha_6 = \{\lambda_8\} & \alpha_7 = \{\#f\} & \alpha_8 = \{\lambda_8\} \\ \alpha_9 = \{\#f, \lambda_8\} & \alpha_{10} = \{\lambda_{10}\} & \alpha_{11} = \{\#f, \lambda_8\} & \alpha_{12} = \{\#f\} \\ \alpha_i = \{\lambda_8\} & \alpha_x = \{\#f, \lambda_8\} & \alpha_y = \{\#f\} & \end{array}$$

2. Les justifications que je donne sont parfois minimalement courtes.

- (a) Incorrect. Les deux expressions sont indifférenciables si on se borne à les appeler. Toutefois, on peut les distinguer grâce à un test de types si ‘f’ n’est pas une fonction.
- (b) Incorrect. Il faut quand même évaluer  $e_{34}$  au cas où elle déclenche une erreur ou une boucle à l’infini.
- (c) Correct. Il montrer que toute évaluation (suite de  $\beta$ -réductions) du programme original est convertible en une évaluation équivalente du programme modifié.
- (d) Incorrect. Bien sûr, la construction de la liste infinie ne se termine jamais donc on ne peut pas en extraire quoi que ce soit. Notez que dans un langage *paresseux*, la transformation serait correcte.
- (e) Incorrect. Le fait de pas évaluer  $e_{81}$  dans certains cas peut faire en sorte qu’une erreur ou une boucle à l’infini est évitée, modifiant la sémantique du programme.
- (f) Incorrect. Si jamais  $e_{64}$  menait à une boucle à l’infini, on verrait la différence.
- (g) Incorrect. Si jamais  $e_{76}$  menait à une erreur, on verrait la différence.